XXXVII. *A Treatise on Rivers and Canals.* *By* Theod. Aug. Mann, *Member of the Imperial and Royal Academy of Sciences at* Bruſſels; *communicated by* Joſeph Banks, *Eſq. P. R. S.*

Read June 24, 1779.

TO JOSEPH BANKS, ESQ. P. R. S.

SIR,

YOUR election to the Preſidency of the firſt literary and ſcientific Society in the world, to a chair ſo long and ſo glorioufly occupied by the great NEWTON; joined to the friendſhip you have been pleaſed to honour me with ſince my being firſt known to you: has encouraged me to ſend you ſomething of my compoſition, as the beſt way of expreſſing my ſincere reſpect and attachment to you, and my profound veneration for the illuſtrious Body which has choſen you for its head. Though various circumſtances, by carrying me very early into foreign countries, have made me from my youth almoſt an

an alien to my native ſoil, and put me in a ſituatio. which apparently muſt make me ever remain ſo; yet, neither time nor diſtance could ever weaken, much leſs obliterate, my tender attachment to it, or my ardent wiſhes for its welfare.

Theſe conſiderations will, I hope, merit a favourable acceptance from the Royal Society of the following piece, which I have the honour of addreſſing to you; and an indulgent condeſcenſion for its imperfection in every reſpect, and particularly in point of ſtyle. Five and twenty years abſence from my native country, and the neceſſity of converſing during that time in different foreign languages, muſt unavoidably have filled mine, without my being ſenſible of it, with idioms and expreſſions in no wiſe Engliſh.

As to the ſubject I have undertaken to treat on this occaſion, I was guided in the choice thereof by the motive of ſaying ſomething that might be uſeful to my native country. The great number of extenſive and magnificent canals, which have been cut through almoſt every part of England of late years, for the uſe of internal navigation, and which do honour to the public ſpirit of the nation, merit to be conſidered in a ſcientifical as well as in a commercial light. Their waters have their laws of motion different in many caſes from thoſe of

rivers: they are liable to many accidents which the others are not, and of a different nature. Theſe accidents do not become ſenſible till many years after their conſtruction, and are better prevented in time than remedied when they happen. I have long lived in a country famous for its navigable canals, and have been much employed, under the eyes of the government of it, upon that ſubject. I only mention this to ſhew, that I have not undertaken to treat a ſubject to which I am an utter ſtranger.

There are, moreover, many conſiderations concerning the laws of motion in rivers and canals in general, the velocity of their currents in proportion to the quantity of their declivity, and the means of aſcertaining therefrom the reſpective heights of the interior parts of continents, which merit the attention of a natural philoſopher. I ſhall venture to offer my thoughts and obſervations (ſome of which, I believe, are new) on all theſe ſubjects in the enſuing Diſſertation, which I ſubmit entirely to the judgment of the Royal Society, and ſhall eſteem myſelf happy if I ſucceed in it, ſo as to be of any uſe to my country, and to be able to teſtify, at the ſame time, my profound reſpect and veneration to you, DEAR SIR, and to the illuſtrious body over which you preſide.

SECTION I.

Different uses for which canals are made, with an account of the principal authors who have wrote concerning them.

1. Artificial canals are to be confidered in a double light; as facilitating commerce by means of internal navigation, and as preventing inundations by carrying off the too great abundance of water from low and flat countries, fuch as are Holland, Flanders, &c. In thefe laft named countries they ferve at once for both purpofes; and it is in this double light that I fhall confider them in the enfuing difcourfe. If canals for draining have fluices upon them, particularly at the end whereby they difcharge their waters, as is univerfally the cafe in the Low Countries, they differ in no wife from navigable canals: if they have nothing to fuftain their waters in them, they are to be confidered in every refpect as rivers or rivulets, and follow the fame laws. It muft, therefore, be carefully kept in mind, that whenever I mention canals, I mean thofe only whofe waters are kept up by fluices, and never thofe without them, which I include, without diftinction, under the common appellation of rivers; for they

they are no more than artificial ones. If I miſtake not, all the navigable canals in England are of the firſt ſort; that is, have their waters kept up, and let off by ſluices. This neceſſary diſtinction will take away all ambiguity from what I have to ſay on canals throughout the following diſcourſe.

2. But that I may fulfil the taſk I have undertaken, it is neceſſary firſt of all to lay down ſuch principles on the nature of rivers and canals in general as have been demonſtrated true both by calculation and experience; to the end, that we may deduce from thence the true laws of motion of their waters, and the quantity of declivity of their beds: for this purpoſe, and becauſe a large volume would hardly ſuffice to compriſe all the demonſtrations of theſe principles, which, conſequently, I am obliged to omit in this treatiſe, it will not be amiſs to mention the principal authors who have treated this ſubject in different ages and countries, in whoſe works the demonſtrations of all the principles I ſhall lay down may be found, if any one doubts the truth of them. Theſe are the following.

SEXTUS JULIUS FRONTINUS, de Aquæ-ductibus Urbis Romæ, cum Notis POLENI, impreſſ. 1722.

JOHN BAPTIST ALEOTTI, Hydrometrician to the Duke of Ferrara, and to Pope CLEMENT the VIIIth.

* Don BENEDICT CASTELLI, Benedictine Abbot, de Mensurâ Aquarum Currentium.

J. B. BARATTERI, de Architettura d'Acque, lib. VI. Piacenza, in folio, 1656.

ALEXANDER BELTINZOLI, of Cremona.

NICOLAUS CABEUS, in Libris Meteorum.

GALILEI GALILEO.

* JOH. BAPT. BALIANI, de Motu Liquidorum.

* JOH. BAPT. RICCIOLI, Geographiæ et Hydrographiæ Reform. libro VI. cc. 29. et 30.

* CLAUDE MILLET DESCHALES, de Fontibus et Fluminibus, à prop. 39. usque ad 56.

VARENNIUS, General Geography, with Dr. JURIN's and Dr. SHAW's Notes, edit. of 1765, vol. I. from page 295 to page 358.

Dr. JURIN, in the Philosophical Transactions, N° 355. page 748. et seq.

MARIOTTE, Traité du Mouvement des Eaux.

VARIGNON, Memoires de l'Academie des Sciences de Paris, pour 1699 et 1703.

* Sir ISAAC NEWTON, Princip. Mathem. lib. II. § 7. page 318 et seq. edit. 1726.

* DANIELIS BERNOUILLI, Hydrodynamica, in quarto, Argentorati, 1738.

* DOMENICHE GUGLIELMINI, della Natura de Fiumi, Bononiæ, 1697, in quarto. Ejufdem de Menfurâ Aquarum fluentium, Bononiæ, in quarto.

* JOH. POLENUS, de Caftellis et de Motu Aquæ mixto, Patavii, 1697, 1718, 1723.

* RACCOLTA d'Autori che trattano del Moto dell' Acque, Fiorenza, 1723, 3 vol. quarto, cum fig.

JAC. HERMANNUS, in Phoronomia, cap 10. page 226. et feq.

CHRIST. WOLF, Curf. Mathem. Hydraulicæ, cap. VI. edit. Genevæ, in quarto, 1740.

M. DE BUFFON, fur les Fleuves, dans fon Hiftoire Naturelle, tom. II. p. 38—100. de la 1ere edit. en 12mo.

Several Memoirs upon this Subject in the Collection of the Royal Academy of Sciences of Paris, particularly thofe of M. PITOT, in the volumes for 1730 and 1732.

S'GRAVESANDE, in Elementis Phyficæ, tom. I. lib. II. cap. 10.

* R. P. LECCHI S. J. Hydroftatica, Mediolani, 1765. In this excellent work are feveral pieces by Father BOSCOVICH upon the fame fubject.

* STATTLERI Phyfica, vol. III. p. 232—286. de curfu Fluminum, ejufque Menfuratione et Directione. Aug. Vindel. 1772, 8 vol. in octavo. This author gives many late obfervations and experiments on the motion and meafure

measure of currents, as those of ZENDRINI, HIMENII, &c.

Two other authors have lately wrote upon rivers and canals, but their works are not yet come to my hands; to wit,

Father FRISI, an Italian Barnabite, Professor of Mathematics at Milan.

M. DE LA LANDE, of the Royal Academy of Sciences at Paris, who has just published a History in folio, with plates, of all the Navigable Canals in the World that have come to his Knowledge.

Among the above authors, those marked with an asterisk (*) are they who have treated the subject in question with the greatest exactness or most extent; and it is from them chiefly that I shall lay down such principles and laws of action in rivers and canals as regard the subject I have taken in hand. By this means I shall avoid advancing any thing upon so important a matter, but what is founded upon the most certain and exact experiments, and conformable to what are demonstrated to be the real and unalterable laws of nature.

SECTION II.

The theory of rivers and canals.

I. DEFINITIONS.

3. A river is a greater or leſſer quantity of water which runs conſtantly, by its own gravity, from the more elevated parts of the earth, towards thoſe which are more depreſſed, in a natural bed or channel open above.

4. If this bed or channel is artificial, and has been dug by hands, it is called *a canal*, of which there are two kinds; thoſe where the channel is every where open, and without ſluices, which I call an *artificial river*; and thoſe where the waters are kept up or let off by the means of ſluices: it is this ſecond ſort which I ſhall call henceforwards by the proper name of *a canal.*

5. *A river is ſaid to perſevere in the ſame ſtate* ſo long as there runs off an equal quantity of water in the ſame time, without any increaſe or diminution, ſo that it remains always at the ſame height in the ſame place. When the circumſtances are different from this, it is ſaid reſpectively that *a river increaſes or diminiſhes.*

6. *A ſection* of the bed of a river or canal is a plane drawn perpendicular to the bottom of the bed and to the

direction of the stream of water, and whose limits are those of the water itself which runs off in that place.

7. I call *sections of equal velocity*, all those where the water runs with equal velocity; and *sections of greater or lesser velocity*, those where the water runs faster or slower respectively, and when compared to others.

8. I call *mean velocity of a current or stream of water*, that which a river or canal would have, if all the parts thereof, to wit, those of the bottom, the sides, the middle, and the surface of the same section, ran with an equal velocity, in such a manner that there would pass just as much water in the same time by this uniform motion, as there does now actually pass by the irregular flowing of the stream.

II. PROPOSITIONS, *or laws of action in rivers and canals.*

9. The motion of water in rivers proceeds from the same principle which produces the descent of heavy bodies upon inclined planes.

11. The descent of heavy bodies upon inclined planes follows exactly the same laws as those observed in the descent of heavy bodies in a perpendicular line towards the center of the earth; that is,

1st, They descend by a motion uniformly accelerated.

2dly, The

2dly, The ſpaces, run over by heavy bodies which fall perpendicularly by a motion uniformly accelerated, are in *a duplicate ratio of the times and velocities* reſpectively.

3dly, Theſe ſpaces, in equal times, augment in the ſame ratio as the odd numbers in progreſſion 1, 3, 5, 7, 9, 11, &c.

4thly, Therefore, both the times and the velocities are in a *ſub-duplicate ratio of the ſpaces run over.*

5thly, It is demonſtrated by the principles of mechanics, that the velocity acquired by a heavy body deſcending freely upon an inclined plane, in a given time, is to the velocity which the body would acquire in the ſame time, by falling perpendicularly, as the height of the inclined plane is to its length.

6thly, From whence it follows, that the velocities which bodies acquire in their deſcent upon inclined planes, are *in a direct ratio of the ſquare roots of the quantity of inclination or declivity of the planes.*

11. So that when water flows freely upon an inclined bed, it acquires a velocity, which is always as the ſquare root of the quantity of declivity of the bed.

12. In an horizontal bed, opened by ſluices or otherwiſe, at one or both ends, the water flows out by its gravity alone; and the flowing is quicker or ſlower in a di-

rect ratio of the reſpective heights of the water, by reaſon of the weight of the ſuperior waters upon the inferior.

13. From hence (N° 11, 12.) it follows, firſt, that as much as the declivity of the bed or channel of a river is greater, ſo much alſo will the velocity of the flowing waters be proportionably increaſed.

2dly, As much as the water in an horizontal bed is deeper, ſo much will the velocity of the current be increaſed; and this velocity will diminiſh in proportion to the decreaſing depths of the water in the bed.

3dly, Abſtracting from the reſiſtance cauſed by the bottom and ſides of the bed, as much nearer as the water is to the bottom, ſo much will its motion be accelerated; not only becauſe the inferior waters are more compreſſed by the ſuperior in proportion to their greater depth; but alſo becauſe the inferior ones have a greater declivity than the ſuperior, by reaſon of their greater depth in the bed, where they are more depreſſed with reſpect to the elevation of their common ſource or ſpring. But theſe different velocities of the upper and lower waters in the ſame ſection of the bed (abſtracting from the friction of the bottom and ſides) approximate indefinitely to each other in proportion to the length of the channel, but ſtill without a poſſibility of their ever becoming equal in fact, if they met with no reſiſtance from the bed.

14. There-

14. Therefore, the motion of water flowing freely in an inclined channel, is accelerated by its own weight combined with the quantity of declivity in the bed.

Neverthelefs, the velocity of waters which flow in an inclined bed, during their actual flowing, is not accelerated by the weight which the inferior waters fuftain from the fuperior ones, in cafe the lower parts have already, by the declivity of the bed, a greater velocity than that which the weight of the fuperior ones impreffes upon them. The reafon of which is, that no body which follows another with a leffer velocity can act by impulfion upon that which precedes it with a greater velocity, as is the cafe with regard to thefe fuperior and inferior waters. But the weight of the upper waters begins to accelerate the lower as foon as they fall into an horizontal bed, or one that is fo nearly horizontal as to deftroy the greater velocity of the lower waters above that of the upper.

15. The velocity of rivers depends fometimes upon the fole declivity of their beds; fometimes alfo upon the fole gravity of their waters: and if thefe two caufes fometimes act together, the effect produced is only the refpective excefs of the one above the other. It often happens in the fame fection of a river, that the acceleration of velocity in the inferior parts proceeds from the weight of

the upper waters, while that in the upper parts proceeds from the declivity of the bed.

From whence it follows, that in rivers which have little declivity, it is the depth of the waters which contributes moſt to accelerate their current; and in thoſe whoſe beds have moſt declivity, it is the deſcent of gravity upon an inclined plane which has the greateſt ſhare in producing this acceleration.

To find whether the water in a part of a river where the bed is nearly horizontal flows by the velocity acquired in the preceding declivities, or by the compreſſion of the upper waters upon the lower in that place; a pole muſt be thruſt down to the bottom, and held perpendicular to the current of the water, with its upper end above the ſurface: if the water ſwells and riſes immediately againſt the pole, it ſhews that its flowing is by virtue of a preceding declivity: if, on the contrary, the water ſtops for ſome moments before it begins to riſe againſt the pole, it is a proof that it flows by means of the compreſſion of the upper waters upon the lower.

16. The *abſolute height or elevation* of the ſurface of a river which perſeveres in the ſame ſtate (N° 5.) continually decreaſes, as the diſtance in the river from its ſource increaſes; by reaſon that its bed muſt continually incline and tend towards the center of the earth.

17. The

17. The velocity of each particle of water in a regular channel, that is, where the bed is a regularly inclined plane, may be determined by drawing a perpendicular from the particle propofed to the horizontal curve which paffes through the fpring, or that point of the river where the particle in queftion begins to acquire its velocity. For the velocity which this particle would acquire, in falling freely along the faid perpendicular, is the fame as that which it has acquired in its defcent along the inclined plane of its bed.

18. So long as *a river perfeveres in the fame ftate* (N° 5.) there flows an equal quantity of water in equal times, how unequal foever the fections be through which they flow; and, confequently, where the fection of the river is greater, the velocity of the flowing water is lefs; and where the fection is lefs, the velocity is greater; always in an inverfe proportion. From hence may be deduced the following and other fimilar propofitions.

1ft, Through equal fections, in equal times, and with equal velocity, there muft flow equal quantities of water.

2dly, Through equal fections, in equal times, but with unequal velocities, the quantities of water which flow, are in a direct ratio of the refpective velocities.

3dly, Through

3dly, Through unequal fections, in equal times, and with equal velocities, the quantities of water which pafs are in a direct ratio of their refpective fections.

4thly, Through unequal fections, and with unequal velocities, the quantities of water which flow in equal times, are in a combined ratio of the fections and mean velocities (N° 8.) together.

In a word, the fections of the bed, the mean velocities, the times of flowing, and the quantity which flows, are univerfally in a combined ratio together; and this combination is what is called *the momentum of a river*; and this *momentum of the fame flowing water is univerfally equal*.

19. From hence may eafily be deduced the principles for calculating the quantity of diminution of the water in a lake, pond, or veffel, by any determinate flowings whatfoever: for as the furface of the lake, &c. is to the fection of the current which carries off the waters; fo is the mean velocity of the current in this fection to the decreafe of the waters in the lake, &c. and *vice verfa*.

III. *On*

III. *On the nature of rivers and flowing waters.*

20. Rivers contain divers inherent caufes of the acceleration of their motion.

1ft, Their fprings are either in mountains or on high grounds, and it is by the defcent of the waters from thefe elevations that they acquire a velocity and acceleration of motion fufficient to fuftain and propagate it through the reft of their courfe.

2dly, The cohefion of the particles of a fluid, *in a bed ever fo little inclined*, is a fecond caufe of acceleration of motion in the fluid; becaufe, by their mutual attraction, thofe particles which begin firft to flow draw after them thofe which are contiguous, thefe the following, and fo on *ad infinitum.*

3dly, Moreover, where a river, by flowing in a bed nearly horizontal, has loft a great part of the velocity which it had acquired in the preceding declivities, and the bed by this means is become large and fhallow, which confequently again augments the flownefs of the current; it may, however, recover a part of its velocity, even in the fame horizontal bed, by augmenting the depth thereof, and diminifhing its breadth; for by this means the weight of the fuperior waters upon the infe-

rior is increafed, and confequently the velocity of the whole is augmented (N° 13. 18.). In the fame manner a junction of rivers in the fame bed, by excavating and deepening it, augment the velocity of the common current, as we fhall fhew more particularly hereafter.

21. On the other hand, flowing waters meet with many powerful caufes of refiftance to their motion, which tend continually to diminifh their velocity. Such are the following:

1ft, The attraction and continual friction of the bottom and the fides of the bed, contribute greatly towards retarding the motion of the water.

2dly, The fame effect is produced likewife by the many obftacles which they meet with in their way; fuch as inequalities in the bottom and fides of the channel, banks of fand and mud, rocks, trunks of trees, and other fuch things.

3dly, The many windings and angles made in their courfe, which produce fo much the more refiftance and hindrance to the motion of the water, as the courfe varies more and oftener from a right-line.

4thly, The diminution of their declivity the farther they recede from their fprings; this being generally the leaft towards their mouths, which are for the moft part in extenfive plains.

Finally,

Finally, the natural cohefion of the particles of water *in an horizontal bed* contributes to retard its motion precifely by the fame force which contributes to accelerate it *in an inclined bed*. By diminifhing or taking away the above obftacles to the free motion of water in rivers and canals, the velocity of their currents will be increafed in the fame proportion, and thereby alfo all the dangers and ravages of inundations may be prevented, as we fhall fhew hereafter.

But if fome or all of thefe caufes, in a greater or leffer degree, did not exift in rivers of confiderable depth and declivity, it is demonftrated[(a)], that the velocity of their currents would be accelerated to twelve, fifteen, and, in fome cafes, even to twenty times more than it is at prefent in the fame rivers, whereby they would become ababfolutely unnavigable.

22. The waters in a river or open canal have their motion accelerated, fo long as the effects proceeding from gravitation, declivity, depth, in a word, fo long as *the fum of accelerations furpaffes the fum of refiftances.*

When thefe different fums become equal to each other, the motion of the water is neither accelerated nor retarded, but remains equal, till fomething anew deftroys the *equilibrium*.

(*a*) Vide LECCHI, Hydroftat. et STATTLERI, Phyfic. tom. III. p. 252.

 When

When the ſum of reſiſtances and cauſes of retardation is greater than the ſum of accelerating cauſes, the velocity of the river is diminiſhed in proportion to the exceſs.

23. The *percuſſion* of the waters of a river againſt an obſtacle which is oppoſed to their motion, is the action of the waters ſtriking againſt that obſtacle; and the principles for calculating the quantity of this percuſſion, or the effects which any obſtacles whatever produce in the motion of rivers, by known forces, and in determinate times, are as follows:

1ſt, The percuſſion of the water of a river againſt any obſtacle whatever is univerſally in a compound ratio of the quantity of the plane or planes which the obſtacle oppoſes to the current of water, of the ſine of the angle of incidence which the direction of the current makes with theſe planes, and of the ſquare of the velocity of the ſaid current.

2dly, The reſiſtance, therefore, which the bed of a river oppoſes to its current from any particular obſtacles in it, is in a compound ratio of the magnitude and ſituation of the planes of theſe obſtacles, together with the ſquare of the current's velocity in the place where thoſe obſtacles are found.

3dly, The accelerating force of a river, or that by which it ſurmounts the reſiſtance of its bed in any one

place, compared to that in another, is in a compound ratio of the maſs of water and of the velocity of the current in thoſe places reſpectively.

24. In different parts of the ſame river, the velocity of the current is greater in a direct proportion of the greater declivity of the bed; becauſe the relative gravity of the flowing particles augments in that ratio.

25. But in the ſame ſection of a river, the ſuperior parts, and thoſe which are fartheſt from the bottom and the ſides, will continue their courſe by the ſole cauſe of the declivity of their bed, *how little ſoever it be*; becauſe theſe waters not being retarded by the friction of the bottom and ſides of the bed, or hardly by any other obſtacle whatever, *the leaſt poſſible deviation from a level will produce a current*. But the waters at the bottom of a river, both becauſe of their friction againſt it, and of the irregularities which are almoſt every where found in it, will loſe that little motion which a very ſmall declivity can give them, and their motion in that caſe will be produced alone by the compreſſion of the ſuperior waters upon them.

The inferior waters which thus acquire their motion from the weight of the ſuperior ones upon them, communicate reciprocally a part of their motion, by means of the natural coheſion of the particles together (N° 20.)

 to

to the ſuperior ones, which in an horizontal bed, without this cauſe, would have no other motion than that which is impreſſed upon them by the impulſive force of the waters deſcending from their elevated ſprings.

From whence it appears, that the ſuperior and inferior waters in a river communicate reciprocally a part of their motion to each other; but this can never go beyond a certain point or *maximum*, which is always proportionable to the *momentum* of the river in that place (N° 18.).

It follows from hence, that the greateſt velocity of a river, running in a right-line, is in *the center of its ſection* (N° 6.); that is to ſay, in that point which is the fartheſt poſſible from the ſurface of the water and from the bottom and ſides of the bed, all taken together. This part has the advantage of one half of the depth of water preſſing upon it, and it is exempt from the friction of the bottom and ſides of the bed which are there overcome and vaniſh by the perpendicular compreſſion.

On the contrary, the leaſt velocity of the water is at the bottom and ſides of the bed, becauſe it is there that the reſiſtance produced by friction is greateſt, from whence it is communicated to the other parts of the ſection in *an inverſe duplicate proportion of the diſtances from the bottom and ſides combined together*, until it becomes a

negative

negative quantity, where the effect vaniſhes, or is reduced to nothing.

26. The beſt and moſt ſimple method of meaſuring the velocity of the current of a river or open canal, that I know of, is the following:

Take a cylindrical piece of dry, light wood, and of a length ſomething leſs than the depth of the water in the river: round one end of it let there be ſuſpended as many ſmall weights as may be neceſſary to keep up the cylinder in a perpendicular ſituation in the water, and in ſuch a manner that the other end of it may juſt appear above the ſurface of the water. Fix to the center of that end which appears above water a ſmall and ſtraight rod, preciſely in the direction of the cylinder's axis; to the end, that when the inſtrument is ſuſpended in the water, the deviations of the rod from a perpendicularity to the ſurface of it may indicate which end of the cylinder advances the faſteſt, whereby may be diſcovered the different velocities of the water at different depths; for if the rod inclines forwards according to the direction of the current, it is a proof that the ſurface of the water has the greateſt velocity; but if it inclines back, it ſhews that the ſwifteſt current is at the bottom; if it remains perpendicular, it is a ſign that the velocities at the ſurface and bottom are equal.

This

This inſtrument being placed in the current of a river or canal receives all the percuſſions of the water throughout the whole depth, and will have an equal velocity with that of the whole current *from the ſurface to the bottom* at the place where it is put in, and by that means may be found, both with eaſe and exactneſs, the mean velocity of that part of the river for any determinate diſtance and time.

But to obtain the mean velocity of the whole ſection of the river, the inſtrument muſt be put ſucceſſively both in the middle and towards the ſides, becauſe the velocities at thoſe places are often very different from each other. Having by this means found the *difference of time required for the currents to run over an equal ſpace*; or, *the different diſtances run over in equal times*, *the mean proportional* of all theſe trials, which is found by dividing the common ſum of them all by the number of trials, *will be the mean velocity of the river or canal.*

If it be required to find the velocity of the current only at the ſurface, or at the middle, or at the bottom, a ſphere of wood, of ſuch a weight as will remain ſuſpended in equilibrium with the water at the ſurface or depth which we want to meaſure, will be better for the purpoſe than a cylinder, becauſe it is only affected by the

water

water of that ſole part of the current where it remains ſuſpended.

It is very eaſy to guide both the cylinder and the globe in that part which we want to meaſure, by means of two threads or ſmall cords, which two perſons muſt hold and direct, one on each ſide the river; taking care at the ſame time neither to retard nor accelerate the motion of the inſtrument.

Several other methods have been invented for determining the velocity of the currents of rivers and canals, which may be ſeen in moſt of the authors enumerated in the beginning of this eſſay (N° 2.)

IV. *Application of the preceding laws of the acceleration and retardation of currents to rivers and canals in general, from whence are deduced the various means of preventing or remedying the defects and inconveniencies which muſt neceſſarily happen to them in a ſeries of years.*

27. By combining together all we have ſaid hitherto upon the nature and theory of motion in rivers, and particularly in the articles 13. 18. 20. 21. and 23 it follows evidently, that *the deeper the waters are in their bed in proportion to its breadth, the more their motion is accelerated;*

celerated; ſo that *their velocity increaſes in an inverſe ratio of the breadth of the bed, and alſo of the greatneſs of the ſection*; from whence are deduced the two following univerſal practical rules:

1ſt, To augment the velocity of water in a river or canal, without augmenting the declivity of the bed, we muſt *increaſe the depth and diminiſh the breadth of its bed.*

2dly, But to diminiſh the velocity of water in a river or canal, we muſt, on the contrary, *increaſe the breadth and diminiſh the depth of its bed.*

The above propoſition is perfectly conformable to obſervation and experience; for it is conſtantly ſeen, that the current is the ſwifteſt where the waters are deepeſt and the breadth of the bed the leaſt; and that they flow ſloweſt where their depth is the leaſt and the breadth of the bed the greateſt. "The velocity of waters," ſays M. DE BUFFON [(b)], "augments in the ſame proportion as "the ſection of the channel through which they paſs "diminiſhes, *the force of impulſion from the back-waters "being ſuppoſed always the ſame.* Nothing," continues he, "produces ſo great a diminution in the ſwiftneſs of "a current as its growing ſhallow; and, on the contrary, "the increaſe of the volume of water augments its

(*b*) Hiſt. Nat. tom. II. p. 53. 60. edit. in 12mo.

"velocity more than any other cauſe whatever. The celebrated WOLF, in his Hydraulics[c], aſſures us, that "it is a conſtant and univerſal practice, for accelerating "the current of waters, to deepen the bed, and at the "ſame time to render it narrower."

28. When the velocity which a river has acquired by the elevation of its ſprings and the impulſe of the back-water, is at laſt totally deſtroyed by the different cauſes of reſiſtance which we have enumerated above (N° 21.) becoming equal or greater than the firſt, the bed and current at the ſame time being exactly horizontal, nothing elſe remains to propagate the motion, except *the ſole perpendicular compreſſion of the upper waters upon the lower, which is always in a direct ratio of their depth.* But this neceſſary reſource, this remaining cauſe of motion in rivers, augments in proportion as all the other diminiſh, and as the want of it increaſes: for as the waters of rivers in extenſive plains loſe the acceleration of motion acquired in their deſcent from their ſprings, their quantity accumulates in the ſame bed by the junction of ſeveral ſtreams together, and their depth increaſes in conſequence thereof. This junction and ſucceſſive accumulation of many ſtreams in the ſame bed, which we ſee univerſally in a greater or leſs degree in all rivers throughout the known world, and which is ſo abſolutely

(c) N. 224.

neceſſary to the motion of their waters, can only be attributed, ſays Signor GUGLIELMINI [d], to the infinite wiſdom of the ſupreme Author of Nature.

29. The velocity of flowing waters is very far from being in proportion to the quantity of declivity in their bed: if it was, a river whoſe declivity is uniform and double to that of another, ought only to run with double the ſwiftneſs when compared to it; but in effect it is found to have a much greater, and its rapidity, inſtead of being only double, will be triple, quadruple, and ſometimes even more: for its velocity depends much more on the quantity and depth of the water, and on the compreſſion of the upper waters on the lower, than on the declivity of the bed. Conſequently, whenever the bed of a river or canal is to be dug, the declivity muſt not be diſtributed equally throughout the whole length; but, to give a ſwifter current to the water, the declivity muſt be made much greater in the beginning of its courſe than towards the end where it diſembogues itſelf, and where the declivity muſt be almoſt inſenſible, as we ſee is the caſe in all natural rivers; for when they approach near the ſea their declivity is little or nothing, yet they flow with a rapidity which is ſo much greater, as they contain a greater volume of water: ſo that in

(*d*) Della Natura de Fiumi.

great

great rivers, although a large extent of their bed next the sea should be absolutely horizontal, and without any declivity at all, yet their waters do not cease to flow, and to flow even with great rapidity, both from the impulsion of the back waters, and from the compression of the upper waters upon the lower in the same section.

30. Whoever is well acquainted with the principles of the higher geometry, will easily perceive that it would be no difficult matter so to dig the bed of a canal or river, that *the velocity of the current should be every where equal.* It would be only giving it the form of a curve along which a moving body should recede from a given point, and *describe spaces every where proportional to the times*, allowance being made therein for the quantity of effect of the compression of the upper waters upon the lower. This curve is what is called the *Horizontal Isochronic*, being the flattest of an infinity of others which would equally answer the problem where fluids were not concerned. Upon these curves may be seen LEIBNITZ, HUYGHENS, and the two BERNOUILLI's, who were the first that determined and analysed them, and also many succeeding geometricians, if any one is desirous to occupy himself in such speculations as are more curious than useful, which is not my purpose in this treatise.

 31. Not-

31. Notwithſtanding all we have ſaid concerning the neceſſity of augmenting the depth of a river in a greater proportion than its breadth, if we would accelerate its current; yet it is certain, that this can only be done to a certain point, without deſtroying that equilibrium which ought to reign between the depth and the breadth of the ſection of the ſtream, and thereby putting the river into a ſtate of continual violence, which will inceſſantly exert itſelf to the deſtruction of the banks and wiers made to keep it in, and that action will always exert itſelf in a direct ratio of the greater or leſs want of equilibrium, as it would be eaſy to demonſtrate by the principles of hydraulics. Theſe ſame principles give likewiſe the juſt proportions of this equilibrium between the perpendicular and lateral compreſſion of the water in any river or canal whatſoever, which vary in an inverſe proportion, according to the different degrees of the declivity and velocity of the current; and in a direct one of the greater or leſs coherence and hardneſs of the ſubſtances which compoſe the bed. Rivers which flow in beds compoſed of homogeneous matter of little conſiſtency, ſuch as ſand, &c. are always more broad than deep, when compared to thoſe which run in beds of matter of greater tenacity. It is manifeſt, that the equilibrium here ſpoken of is real,

4 becauſe

becaufe rivers remaining in the fame ftate only widen their beds to a certain pitch which they do not furpafs.

32. M. DE BUFFON remarks, "That people accuf-
"tomed to rivers can eafily foretell when there is going "to be a fudden increafe of water in the bed from floods "produced by fudden falls of rain in the higher countries "through which the rivers pafs. This they perceive by "a particular motion in the water, which they exprefs "in their dialect, by faying that *the river's bottom moves*; "that is, the water at the bottom of a channel runs off "fafter than ufual; and this increafe of motion at the "bottom of the river always announces a fudden in-"creafe of water coming down the ftream. Nor does "their opinion therein," continues the fame author, "feem to be ill-grounded on the nature of things; for "the motion and weight of the waters coming down, "though not yet arrived, muft act upon the waters in "the lower parts of the river, and communicate by im-"pulfion part of their motion thereto; fince a canal or "river contained in its bed is to be confidered in fome "degree as a column of water contained in a long tube, "where the motion is communicated at once throughout "the whole length." In a river or canal, open above, it is only communicated to a certain diftance; that is, as far as the impulfive force of the new increafe and fuperior

rapidity

rapidity of the back-waters acts upon the ſtream, which will always be as far as till this force is gradually, and at laſt wholly, deſtroyed by the ſuperior gravitation of the ſuper-incumbent waters in the ſtream. Something of the ſame kind happens when a very great additional weight comes ſuddenly upon the ſurface of a river or canal; for inſtance, by the launching of a ſhip or of ſeveral boats together upon it. Theſe cauſes increaſe the velocity of the water in the lower parts of the bed, and moreover retard its motion at the ſurface, which effect may properly be called *making the river's bottom move.* For the ſame reaſon, the increaſe of weight of the waters in a ſudden flood, as well as the increaſe of their impulſive force, muſt contribute to produce this effect, and, by increaſing the motion in the bottom of the river, may hinder, for ſome ſpace of time, the ſtream from ſenſibly riſing in the bed.

33. All obſtacles whatever in the bed of a river or canal, ſuch as rocks, trunks of trees, banks of ſand and mud, &c. muſt neceſſarily hinder proportionably the free running off of the water; for it is evident, from what we have ſaid, that the waters ſo far back from theſe obſtacles, until the horizontal level of the bottom of the bed becomes higher than the top of the obſtacles, muſt be intirely kept up and hindered from running off in proportion

portion thereto (N° 23.). Now as the waters muſt continue to come down from their ſources, if their free running off is hindered by any obſtacles whatever, their relative height back from them muſt neceſſarily be increaſed until their elevation, combined with the velocity of their current proceeding from it, be arrived to ſuch a pitch at the point where the obſtacles exiſt, as to counterbalance the quantity of oppoſition or impediment proceeding from thence, which frequently does not happen until all the lower parts of the country round about are laid under water.

34. Now it is certain from all experience, that the beds of rivers and canals in general are ſubject to ſome or others of the obſtacles above mentioned. If rocks or trees do not bar their channels, at leaſt the quantity of ſand, earth, and mud, which their ſtreams never fail to bring down, particularly in floods, and which are unequally depoſed according to the various windings and degrees of ſwiftneſs in the current, muſt unavoidably, in courſe of time, fill up, in part, different places in the channel, and thereby hinder the free running off of the back waters. This is certainly the caſe, more or leſs, in all rivers, and in all canals of long ſtanding, as is notorious to all thoſe well acquainted with them. Hence, if theſe accidents are not carefully, and with a

conſtant

conſtant attention prevented, come inundations, which ſometimes lay waſte whole diſtricts, and ruin the fineſt tracts of ground, by covering them with ſand: hence rivers become unnavigable, and canals uſeleſs, for the purpoſes for which they were conſtructed. Canals, in particular, by reaſon that their waters for the moſt part remain ſtagnant in them, are ſtill more liable than rivers to have their beds fill up by the ſubſiding of mud, and that eſpecially for ſome diſtance above each of their ſluices; inſomuch, that if continual care is not taken to prevent it, or remedy it as often as it happens, they will ſoon become incapable of receiving and paſſing the ſame veſſels as formerly. Nay, the very ſluices themſelves, if the floors of their bottoms are not of a depth conformable to the bed of the canal, will produce the ſame accidents as thoſe we have been ſpeaking of; for if they are placed too low, they will be continually filling up with ſand or mud; if too high, they have the ſame effect as banks or bars in the bed of a river, that is, they hinder all the back-waters under their level from running off, and ſoon fill up the bed to that height by the ſubſiding of mud. This effect is much accelerated by the ſhutting of the lower ſluices, which makes a great volume of water reflow back to thoſe next above them, till the whole is filled and becomes ſtagnant. Now it is evident,

that

that this ſtate of things muſt contribute far more to the ſubſidency of mud and all other matters brought down by the waters in canals, than can be the caſe in rivers whoſe currents conſtantly flow.

35. I do not ſuppoſe that theſe inconveniencies can have yet manifeſted themſelves by any very ſenſible effects in the many new canals and ſluices lately conſtructed in England; but as the ſame cauſes do not ceaſe to act more or leſs every where, the effects which neceſſarily follow from them will likewiſe become more and more ſenſible, unleſs continual care be taken to prevent them. The waters of all rivers and canals are from time to time muddy: their ſtreams, particularly during rains and floods, carry along with them earth and other ſubſtances which ſubſide in thoſe places where their currents are the leaſt, whereby their beds are continually raiſed: ſo that the ſucceſſive increaſe of inundations in rivers, and of unfitneſs for navigation in canals, when they are neglected and left to themſelves, is a natural and neceſſary conſequence of the ſtate of things, which no intelligent perſon can be at a loſs to account for; and yet I have known whole countries remain in this habitual ſtate of negligence to their very great detriment.

36. Having thus ſhewn the principal accidents which rivers and canals are liable to, with the cauſes of them, I

 ſhall

ſhall proceed to point out the moſt efficacious methods of preventing them, or at leaſt of diminiſhing their effects. Perhaps it would have been more proper to have deferred doing this till I ſhould have ſaid all I have to ſay upon the nature of rivers and canals: however, I ſhall forego the more ſcientific order of things, for the ſake of bringing the means of remedying the accidents and inconveniences which happen, nearer to the cauſes that produce them, whereby their connexion and efficacity may be better judged of. For this end, I ſhall here lay down, briefly and in general terms, the methods moſt proper for the purpoſe in queſtion. They flow immediately from the principles already laid down in this eſſay, and do not need many words to make them compleatly underſtood.

37. A work of this kind, if it is properly conducted, muſt be begun at the lower end of the river or canal; that is to ſay, at that end where their waters are diſcharged into the ſea, or where they fall into ſome other greater river or canal, from whence their waters are carried off without farther hindrance. If it is a river whoſe bed, by being filled up with mud, ſand, or other obſtacles, and by being otherwiſe become irregular in its courſe, is thereby often ſubject to inundations, and incapable of internal navigation, the point, from which

the work muſt be begun and directed throughout all the reſt of the channel, is from the loweſt water-mark of ſpring tides on the ſhore at the mouth of the river; or even ſomething below it, if it can be done; though this part will ſoon fill up again by the ſand, mud, &c. which the tides ceaſe not to roll in.

If it is a canal whoſe bed is be to dug anew, or one already made, which is to be cleaned and deepened from the ſea ſhore or ſome large river back into the country, and where no declivity is to be loſt; as is the caſe in all flat countries: the work muſt be begun, and the depth of the whole channel directed, from the low water-mark of ſpring tides, if the mouth is to the ſea, or from ſuch a depth in the channel of the river, if the canal falls into one, that there may be ſuch a communication of water from the canal to the river, in all ſituations of the current, as may let boats freely paſs from one to the other. This, of courſe, muſt alſo direct the depth of the floor of the laſt ſluice towards the mouth of the canal, be it to the ſea or into a river. If the bottom or floor of a ſluice already conſtructed be too low, it will ſoon fill up with ſand or mud, and thereby hinder the gates from opening, unleſs it be continually cleaned out; if, on the contrary, this floor be too high, and in a canal whoſe natural declivity is too little for the free current of the water, as is

generally the cafe in Holland and Flanders, all depth of the bed of the canal below the horizontal level of the bottom of the fluice will ferve to no manner of purpofe, either for navigation, or for carrying off the back-waters, but will foon fill up with mud, in fpite of all means ufed to the contrary, except that of digging it continually anew to no manner of purpofe; as is evident from the reafons given above (N° 33. 34.).

38. Setting off from this determinate point, at the mouth of a river, or at the bottom of the laft fluice upon a canal, which are to be cleaned and deepened; the work muft be carried on, in confequence, uniformly throughout their whole courfe backwards into the country as far as is found neceffary for the purpofes intended. This is to be done after the following manner:

1ft, One muft dig up and carry away all irregularities in the bottom and fides of the bed, fuch as banks of fand and mud, rocks, ftumps or trunks of trees, and whatever elfe may caufe an obftacle to the regular motion of the water, and to the free paffage of veffels upon it.

2dly, If the declivity of the bed fhould be ftill too little to give a fufficient current to carry off the water as often and as faft as is neceffary, the whole bed itfelf muft be regularly deepened, and what is dug out from the bottom muft be laid upon the fides, to render it narrower in proportion

proportion to its depth. The reaſon of this is evident from all that has been ſaid.

3dly, Wherever the banks are too low to contain the ſtream in all its ſituations, they muſt be ſufficiently raiſed; which may be conveniently done with what is dug out from the bed: and the whole being covered with green turf will render theſe banks firm and ſolid againſt the corroſion of the water. It is proper at all times to lay upon the banks what is dug from the bed, by which they are continually ſtrengthened againſt the force of the current.

4thly, It is often neceſſary to diminiſh the windings and ſinuoſities in the channel as much as poſſible, by making new cuts whereby its courſe may approach towards a right line. This is a great reſource in flat countries ſubject to inundations; becauſe thereby all the declivity of a great extent of the river, through its turns and windings, may be thrown into a ſmall ſpace by cutting a new channel in a ſtraight line; as may generally be done without obſtacle in ſuch countries as I am ſpeaking of, and hereby the velocity of the current will be very greatly augmented, and the back-waters carried off to a ſurprizing degree, as is evident from what is ſaid above in N° 29.

5thly,

5thly, Wherever there is a confluence of rivers or canals, the angle of their junction muſt be made as acute as poſſible, or elſe the worſt of conſequences will ariſe from the corroſion of their reſpective ſtreams; what they carry off from the ſides will be thrown into irregular banks in the bottom of the bed. This acute angle of junction may always be procured by taking the direction at ſome diſtance from the point of confluence.

6thly, Wherever the ſides or banks of a river are liable to a more particular corroſion, either from the confluence of ſtreams, or from irremediable windings and turns in the channel, they muſt be ſecured againſt it as much as poſſible by *weirs:* for this corroſion not only deſtroys the banks, and alters by degrees the courſe of the river, but alſo fills up the bed, and thereby produces all the bad effects we have ſpoken of above in N° 33. 34. &c.

7thly, But the principal and greateſt attention in digging the beds of rivers and canals muſt be had to the *quantity and form of their declivity.* This muſt be done uniformly throughout their whole extent, or ſo much of it as is neceſſary for the purpoſes in hand, according to the principles laid down above (in N° 29 and 30.) Conformable thereto, the depths of their beds, and of the floors of their ſluices, at the mouths whereby they diſ-

charge

charge their waters, being fixed according to what we have ſaid in N° 37. the depth of the reſt of the beds, and the quantity of declivity therein, muſt be regulated in conſequence thereof, ſo as to increaſe regularly the quantity of declivity in equal ſpaces the farther we recede from their mouths, and proceed towards their ſources or to the part where the regular current is to take place.

If the depth and volume of water in a river or canal is conſiderable, it will ſuffice, in the part next the mouth, to allow one foot perpendicular of declivity through ſix, eight, or even, according to DESCHALES [d], ten thouſand feet in horizontal extent; at moſt it muſt not be above one in ſix or ſeven thouſand. From hence the quantity of declivity in equal ſpaces muſt ſlowly and gradually increaſe as far as the current is to be made fit for navigation; but in ſuch a manner, as that at this upper end there may not be above one foot of perpendicular declivity in four thouſand feet of horizontal extent. If it be made greater than that in a regular bed containing a conſiderable volume of water, the current will be ſo ſtrong as to be found very unfit for the purpoſes of navigation, as will appear hereafter, when I come to inveſtigate the quantity of declivity in ſeveral rivers, the degree of ſwiftneſs of whoſe currents is well known.

(*d*) De Fontibus et Fluviis, prop. 49.

39. I

39. I dare boldly affirm, from the certain principles of hydrodynamics laid down in this eſſay, that if the above mentioned things (N° 37. 38.) were carried into execution in a proper manner; the velocity of currents and the acceleration of motion of the waters in rivers, and in canals when their ſluices are open, might be increaſed to any degree that can be required for opening their beds, and for preventing inundations during great rains or ſudden floods: by carrying off more ſwiftly the great acceſſion of water which then takes place. It would not be difficult, by theſe means, to increaſe the velocity of the current to double and triple what it is in rivers and canals, whoſe beds for a long ſpace of time have been left to themſelves. There is not, perhaps, a country on earth but what might be freed from inundations by theſe means. But it may be objected, that if all I have adviſed was put in execution, even in the flatteſt countries, the currents of rivers (for canals ſhut up with ſluices are here out of the queſtion) would become incommodious, if not unfit, for navigation, eſpecially againſt their ſtreams. This objection would be of weight if it was not evident that the various means which I have pointed out may be executed in whole or in part, to a certain degree, and no farther than neceſſary for the purpoſes required. But, as it is certain that a ſtrong and

regular

regular current in a river is the beſt of all means for keeping it open and deep, and for preventing the formation of banks in the bed by the ſubſidency of mud, &c. which it does not allow time to precipitate; I leave it to be conſidered, whether it is better to have a free and open navigation ſomething incommoded by the ſtrength of the current, or to have ſoon no navigation at all, without repeatedly digging the bed anew.

40. I ſhall not here enter into the mechanical part of the methods of digging and cleaning canals, rivers, and ſea ports, or into any deſcription of the machines and inſtruments neceſſary for that purpoſe. The ſubject would lead me much too far: beſides all theſe things may be found much at length in moſt of the authors who have wrote upon hydraulic-architecture, ſuch as BARATTERI, CORNELIO MEYERI, GUGLIELMINI, and *a notorious anonymous French plagiary*, who has taken from MEYERI, without ever naming him, almoſt all that is contained in his book, publiſhed at Paris in 1693, and at Amſterdam in 1696, in octavo, under the title of *Traité des Moyens de rendre les Rivières navigables*. But the author who has treated this ſubject with the greateſt care, and moſt at length, is the celebrated BELIDOR, in his *Architecture Hydraulique*, 4 vol. in quarto. To theſe may be added a late memoir of M. FORFAIT of

Rouen, vice-architect of the French navy, which gained the prize of the Royal Academy of Sciences and Belles Lettres of Mantua, for having given the beft folution of a problem propofed by that Society in 1776, in the following terms: "*To indicate the beft and cheapeft method* "*of freeing navigable canals from banks of fand and* "*earth formed in their beds which render them too fhal-* "*low.*" This piece, printed at Mantua, by Pazzoni, in 1778, contains fixty-three pages in quarto, and is divided into two parts; the firft contains the means of preventing the formation of banks in navigable canals; and the fecond offers divers methods for remedying them when they are already formed. For this purpofe the author propofes fix different machines of his own invention: the firft may be employed in rivers near the fea, and fubject to the ebb and flow of the tides; the fecond may be ufed in thofe where the waters are always nearly of the fame height and velocity; the third and fourth are to be ufed in thofe places where the violence of the currents corrode the beds; and the two laft ferve to break up the banks of fand or earth formed in the bottom, and to carry off all heterogeneous bodies funk in the river, which caufe an obftacle to the current. It would be difficult to give a juft idea of thefe machines without the help of the fix plates which accompany the piece;

piece; but as this production of a foreigner has been crowned in Italy, the country of all others in which, from all antiquity, the ſcience of rivers and canals has been moſt cultivated, we cannot well doubt of its merit, or that it is worthy of a tranſlation into our own language.

V. *Other conſiderations on the nature of rivers and inundations.*

41. Rivers flowing along plains, as well as through vallies, have naturally their beds in the loweſt part of the ground comprized between the oppoſite hills or mountains: nevertheleſs, the ſurface of the water of a river in the midſt of a plain is often higher than the ſurface of the grounds adjacent to the banks of the river. This proceeds from the continual ſubſiding of the mud, &c. brought down by the ſtream during floods; the waters in that caſe uſually overflowing the banks ſpread themſelves over the plain, where they loſe a great part of the ſwiftneſs of their current, which contributes greatly to the ſubſiding of the mud they contain; ſo that the farther they flow upon the plain, the clearer they grow, and the leſs remains to ſubſide. From hence the greateſt precipitation of mud muſt be in the parts of the plain neareſt the ſides of the river, which in length of time will raiſe

theſe grounds above the reſt of the plain. Again, the waters in the bed itſelf depoſing inceſſantly a part of the mud, &c. brought down by the ſtream, muſt continually, though inſenſibly (for a long ſpace of time) raiſe the channel and banks of the river above the reſt of the plain. Theſe cauſes may at laſt contribute to the forming of an intire new bed for the river; for as all rivers carry down in their ſtreams more or leſs mud and other heterogeneous matters, which do not ſubſide regularly in all parts alike, but muſt precipitate faſteſt where the current is ſloweſt; there muſt accumulate by little and little in theſe parts ſuch banks of ſand and mud, as will in time hinder the current of the waters, make them reflow, and at laſt totally change their direction.

Canals are ſtill more ſubject than rivers to have their beds raiſed and their currents ſtopped by the ſubſiding of mud and heterogeneous matter in different places, and eſpecially juſt above their ſluices; becauſe of the ſudden ſtagnation of the water which firſt begins there as often as the ſluices are ſhut: and as there is a neceſſity for keeping them for the moſt part ſhut, the ſtagnating waters in their beds muſt precipitate their mud, &c. in a much greater proportion than can be done in the currents of rivers, which are in a continual motion towards the ſea.

42. I

42. I call *center of the current*, or, more properly, *line of greateſt current*, that *line which paſſes through all the ſections of a river, in the point where the velocity of the current is the greateſt of all.* We have ſeen above (N° 25.) that if the current of a river is regular, and in a right line, its center or line of greateſt velocity will be preciſely in the center of the ſections (N° 6.): but, on the contrary, if the bed is irregular and full of turns and windings, the center or line of greateſt current will likewiſe be irregular, and often change its diſtance and direction with regard to the centers of the ſections through which the waters flow, approaching ſucceſſively, and more or leſs, to all parts of the bed, but always in proportion and conformably to the irregularities in the bed itſelf.

This deviation of *the line of greateſt current* from the centers of the ſections through which it paſſes, is a cauſe of many and great changes in the beds of rivers, ſuch as the following:

1ſt, In a ſtraight and regular bed, the greateſt corroſion of the current will be in the middle of the bottom of the bed; becauſe it is that part which is neareſt to the line of greateſt current, and at the ſame time which is moſt acted upon by the perpendicular compreſſion of the water. In this caſe, whatever matters are carried off from the bottom will be thrown, by the force of the current,

current, equally towards the two ſides, where the velocity of the ſtream is the leaſt in the whole ſection.

2dly, If the bed is irregular and winding, the line of greateſt current will be thrown towards one ſide of the river, where its greateſt force will be exerted in proportion to the local cauſes which turn it aſide: in ſhort turns of a river there will be a gyration, or turning round of the ſtream, by reaſon of its beating againſt the outer ſide of the angle; this part will be corroded away, and the bottom near it excavated to a great depth. The matters, ſo carried off, will be thrown againſt the oppoſite bank of the river where the current is the leaſt, and produce a new ground, called an *alluvion.*

3dly, Inequalities at the bottom of a river retain and diminiſh the velocity of the water, and ſometimes may be ſo great as to make them reflow: all theſe effects contribute to the ſubſiding of ſand, earth, and other matters thereon, which ceaſe not to augment the volume of the obſtacles themſelves, and produce ſhallows and banks in the channel. Theſe in time, and by a continuance of the cauſes, may become iſlands, and ſo produce great and permanent changes and irregularities in the beds of rivers.

4thly, *The percuſſions of the center of the current* againſt the ſide of the bed are ſo much the greater as they are made

made under a greater angle of incidence; from whence it follows, that the force of percuſſion, and the quantity of corroſion and of detriment done to the banks and weirs of rivers, and to the walls of buildings made therein, and which are expoſed to that percuſſion, *are always in a direct compound proportion of the angle of incidence, of the greatneſs and depth of the ſection together, and of the quantity of velocity of the current.*

5thly, It may happen in time, that the excavation of the bottom, and the corroſion of the ſides, will have ſo changed the form of the bed as to bring the force of percuſſion into equilibrium with the velocity and direction of the current; in that caſe, all farther corroſion and excavation of the bed ceaſes (N° 31.)

6thly, This gives the reaſon why when one river falls into another almoſt in a perpendicular direction, and makes with it too great an angle of incidence, this direction is changed in time, by corroſions and alluvions, into an angle much more acute, till the whole comes into equilibrium.

7thly, So great and ſuch continued irregularities, from local cauſes, may happen in the motion of a river, as will intirely change its ancient bed, corrode through the banks, where they are expoſed to the greateſt violence of percuſſion

percuſſion of the ſtream, and open new beds in grounds lower than what the old one is become.

8thly, Hereupon the ſtate of the old bed will entirely depend on the quantity of water, and on the velocity and direction of the current in the new one; for immediately after this diviſion of the waters into two beds is made, the velocity of the current in the old one will be diminiſhed in proportion to its leſs depth. In conſequence thereof, the waters therein will precipitate more of their mud, &c. in equal ſpaces than they did before; which will more and more raiſe up the bottom, ſometimes even till it becomes equal with the ſurface of the ſtream. In this caſe, all the water of the river will paſs into the new bed, and the old one will remain intirely dry. It is well known, that this has happened to the Rhine near Leyden, and to many other rivers.

9thly, Hence the cauſe of the formation of the new branches and mouth, whereby many great rivers diſcharge their waters into the ſea.

43. But in proportion as *a river, that has none of theſe obſtacles in its bed,* approaches towards its mouth, we ſee the velocity of its current augment, at the ſame time that the declivity of the bed diminiſhes, the cauſes of which have been explained above (N° 29.). It is for this reaſon, that inundations are more frequent and conſi-

derable, and do more damage in the interior parts of a country, than towards the mouths of most rivers.

In the Po, for example, the height of the banks made to keep in the waters diminishes as the river approaches to the sea. At Ferrara they are twenty feet high; whereas nearer the sea they do not exceed ten or twelve feet, although the channel of the river is not larger in one place than in the other.

44. The mouths of rivers, by which they discharge their waters into the sea, are liable to great variations, which produce many changes in them.

1st, The velocity and direction of the current at these mouths are in a continual variation, caused by the tides, which alternately retard and accelerate the stream.

2dly, During the flowing of the tide, the current of the river is first stopped, then turned into a direction intirely contrary throughout a considerable extent; if we may believe M. DE BUFFON, there are rivers in which the effect of the tides is sensible at 150 or 200 leagues from the sea.

3dly, This state of things is a cause of a great quantity of sand, mud, &c. being precipitated and accumulated in the channel near the mouth. This continually raises and widens the bed, and at last changes it intirely into a new place, or at least opens new mouths to dis-

 charge

charge the waters at. The Rhine, the Danube, the Wolga, the Indus, the Ganges, the Nile, the Miſſiſippi, and many other rivers, are inſtances of this.

4thly, All theſe effects are leſs ſenſible at the mouths of little rivers, as their currents oppoſe no ſenſible obſtacle to the flowing of the tides; ſo that the ebb carries off again what the flow had brought in.

45. Whenever the courſe of a river throughout a conderable extent of country approaches towards a right line, its current will have a very great rapidity; and the velocity wherewith it runs diminiſhing the effect of its natural gravitation, the middle of the current will riſe up, and the ſurface of the river will form a convex curve of ſufficient elevation to be perceived by the eye; the higheſt point of this curve is always directly above *the line of greateſt current* in the ſtream.

On the contrary, when rivers approach near enough to their mouths for a ſenſible effect to be produced in them by the flowing of the tides; and alſo when in other parts of their courſe they meet with obſtacles at the ſides of their channel: in both theſe caſes the ſurface of the water at the ſides of the current is higher than in the middle, even though the ſtream be rapid. In this ſituation of things, the ſurface of the river forms a concave curve, the loweſt point of which, or that of inflexion, is

directly over *the line of greatest current.* The reason thereof is, that there are in this case two different and opposite currents in the river; that whereby the waters flow towards the sea, and preserve their motion therein even to a considerable distance; and that of the waters which remount, either by the flowing of the tide, or by their meeting with local obstacles, which form a *counter current*, so much the more sensible as the flowing of the tide is stronger, or as the percussion of the water is made against greater obstacles, and in a direction nearer to a perpendicular to them. From both these causes, the greater of which by far is that of the tides, the water near the sides of the channel, where the velocity of the descending stream is naturally the least (N° 25), takes a contrary direction, and runs back in the river, while that in the middle continues to flow on towards the sea. This counter current is what the French call *a remous.*

An island in the middle of a river produces the same effect as obstacles at the sides, regard being had to the difference of situation of each.

Eddies and *whirlpools* in rivers, in the center of which there appears a conical or spiral cavity, and about which the water turns with great rapidity and sucks in whatever approaches it, proceed in general from the mutual percussion of these two counter currents; and the vacuity in

the middle is produced by the action of the centrifugal force, whereby the water endeavours to recede, in a direct ratio of its velocity, from the center about which it moves.

46. If rivers perfevered always nearly in the fame ftate (N° 5.) the beft means of diminifhing the velocity of the current, when it is found too great for the purpofes of navigation, would be by widening the canal: but as all rivers are fubject to frequent increafe and diminution, and confequently to very different degrees of velocity and force in the current, this method is liable to produce very detrimental effects; for, when the waters are low, if the channel is very large in proportion, the ftream will excavate a particular bed, which, according to the irregularities of the bottom, will form various turnings and windings with regard to the principal bed; and when the waters come to increafe, they will follow, to a certain degree, the directions which the bottom waters take in this particular bed, and thereby will ftrike againft the fides of the channel, fo as to deftroy the banks and caufe great damages.

It would be poffible to prevent in part the bad effects proceeding from the current ftriking againft the banks, by opening, at thofe places where it ftrikes, little gulfs into the land, dug in fuch a form and direction as that the

ftriking

ſtriking current ſhould enter and circulate therein, ſo as to deſtroy, or at leaſt greatly diminiſh, its velocity. This effect would be felt for a conſiderable way down the river.

This ſame method might probably be uſed with ſucceſs againſt the deſtruction of bridges, weirs, &c. by the violence of the ſtream during floods. Such gulfs being dug into the outer ſide of thoſe turnings in the river which are immediately above the place to be ſecured from the violence of the ſtream, would ſucceſſively diminiſh its velocity, its force and dangerous effects, a conſiderable way down the river. It is true, this method might contribute to produce an overflowing of the river upon the grounds adjacent to thoſe artificial gulfs, this being a natural conſequence of the decreaſe of the velocity of the current in thoſe places; and it would remain to be conſidered whether thoſe local inundations, or the danger of deſtruction of the bridges or edifices in the river, were the leſſer evil.

47. The nature of inundations, and the manner of their formation, merit a particular attention in this place.

While the volume of water in the *bed* of a river increaſes, the velocity of the current increaſes in proportion, as has been repeatedly ſhewn above (N° 13. 18. 20.

23. 27. 28. 29.). But from the moment that part of this water overflows the bed, the velocity thereof begins to diminiſh (N° 41.) and does ſo more and more, the farther it flows and ſpreads on the plain. So that the overflowing being once begun, it is a natural conſequence, that the inundation ſhould continue for ſeveral days; for though the volume of water brought down by the flood during that time ſhould decreaſe, yet, as the quantity of what runs off decreaſes likewiſe, from the great decreaſe of velocity in what overflows the plains, it will continue to produce the ſame effect as if the volume of water coming down had not diminiſhed, until the whole of the ſtream be every where contained again within the bed of the river. When that is become the caſe, the waters that have overflowed the plain will decreaſe thereon, by gradually and ſlowly running off, and alſo by evaporation, till they wholly diſappear. If this was not ſo, we ſhould ſee rivers overflow for an hour or two, and then return again within their beds, a thing contrary to general obſervation; for we conſtantly ſee inundations, once begun in flat countries, laſt for ſeveral days together, although in the mean while the rain ceaſes, and the quantity of water coming down diminiſhes. This muſt be the caſe, becauſe as the overflowing diminiſhes the velocity, and conſequently the quantity of water carried

carried off, it has the ſame effect as if a greater quantity ſtill continued to come down.

It may not be uſeleſs to remark here, that what we have often ſaid in this eſſay becomes evident from theſe obſervations on nature, as well as from the principles laid down in it; to wit, that the moſt direct and efficacious method of preventing inundations is by deepening the bed and raiſing the banks of the river.

It may likewiſe be obſerved, with regard to inundations, that if the wind blows directly contrary to the current of the river, the overflowing will be greater than it would have been otherwiſe, becauſe this accident diminiſhes the velocity of the ſtream: but, on the contrary, if the winds blow in the ſame direction with the current of the river, the inundation will be leſs than otherwiſe, and ſooner at an end; becauſe this accidental cauſe augments the velocity of the ſtream.

VI. *On the confluence of rivers, and on the ſeparation of the ſame river into divers branches and mouths, with the effects thereof upon the velocity of currents, inundations*, &c.

48. All great and long rivers receive into their beds many others of different magnitude throughout the extent

tent of their courſe. This is evident to every one who only caſts his eyes over a map. The Rhine and the Po, in particular, receive each above one hundred others great and ſmall; the Danube above two hundred; the Wolga as many; the river of Amazons receives into its vaſt bed a prodigious number, ſome of which are five or ſix hundred leagues in length, and are of ſuch a depth and breadth as would make them elſewhere paſs for capital rivers. M. DE BUFFON (e) gives a liſt of the more conſiderable of thoſe which fall into other great rivers throughout the known world. Many curious particulars may be ſeen in VARENIUS's General Geography, part I. chap. XVI. concerning rivers; but of a nature which does not enter into my plan. The works themſelves are in every body's hands, and may be conſulted by thoſe who pleaſe.

This confluence of rivers is ſo neceſſary for propagating the motion of the water throughout a long courſe, and for renewing and accelerating from time to time its velocity, which otherwiſe would be too greatly diminiſhed by the reſiſtance of ſo many obſtacles as they meet with in their way, that, as we have ſaid above (N° 28.) after Signor GUGLIELMINI, it can only be attributed

(e) Hiſt. Nat. tom. II. p. 75, 76.

to

to the infinite wisdom of the Author of Nature, in the original disposition of things.

49. We have seen above (N° 18. 27. 28.) that the increase of a river or canal by the new waters which it receives, *is universally in an inverse ratio of the new velocity which is acquired therefrom.* If this velocity is greater, the increase of the section of the new stream will be less in proportion, and *vice versâ.* It follows from hence, that it is possible for one river or open canal to fall into another river or open canal of equal magnitude with itself, and yet the section of the current in the common bed after their confluence shall be no greater than it was in each of them before their junction. It is certain that this will be the case as often as *the confluence of the two augments the velocity of the common current in the same proportion with the increase of the volume of waters*; both the greater rapidity of the current, and the greater volume of water in the bed after the junction, serving to deepen it in proportion to its breadth, will contribute towards the above effect. Another cause will likewise add thereto; to wit, that instead of the resistance from the attraction, friction, and other obstacles, in two beds, which give two bottoms and four sides, there are, after the confluence, only those of one bed, of one bottom and two sides. Moreover, the center of the section in the common bed

 is

is farther from the bottom and ſides thereof, than it is in the ſeparate beds. All theſe cauſes, in proportion to their reſpective quantities, contribute to accelerate the velocity of the common ſtream.

50. It is not leſs certain, that in rivers which bring down a great abundance of water, the more the velocity and diſcharge thereof at their mouths are retarded and diminiſhed by the tides, the winds, the rolling in of the ſea, &c. the more will the back-waters increaſe in height, and endanger overflowing the inner parts of the country. This is evident, becauſe the decreaſe of velocity in the current, and the increaſe of height of all the back-waters that are affected thereby, are in a reciprocal inverſe ratio one of another (ſee above N° 32.)

Nature itſelf teaches us a method of preventing, or at leaſt of diminiſhing, this effect. We ſee all great rivers overcharged with a vaſt volume of water divide, when they come near the ſea, into different branches and mouths, whereby the ſuper-abundance of their waters is diſcharged. This is the caſe with the Scheld, the Rhine, the Rhone, the Po, the Danube, the Wolga, the Euphrates, the Indus, the Ganges, the Nile, the Niger, the Oroonoko, the River of Amazons, and with almoſt all other great rivers.

This ſeparation and diſperſion of the too great quantity of water into ſeveral channels is one cauſe of their ſeldom overflowing the country near their mouths (f), becauſe it gives a greater depreſſion and declivity to the ſurface of the current, and thereby facilitates the running down of the waters from the interior parts of the country, foraſmuch as their beds are every where regular and free from obſtacles to their current.

51. Notwithſtanding the apparent oppoſition to what has been ſaid in ſeveral other parts of this treatiſe, I repeat again, that this diviſion and diſperſion of the waters into ſeveral branches and channels *when there is ſuch an abundance of it as is ſufficient to keep up the velocity both in the old and new channels*, augments the declivity, and thereby facilitates the running off of all the back-waters from the inner parts of the country, as far as the bed is regular and free from obſtacles, according to what is laid down above (N° 38.).

But whenever this ſuper-abundance of waters, ſufficient for keeping up the velocity in each channel nearly to what it was before the ſeparation or divarication, ſhall be found wanting, it is certain, that this diviſion and diſperſion of the waters into ſeveral channels will only ſerve to diminiſh the velocity of the current in each, whereby

(f) Another cauſe thereof is pointed out above, N° 43.

as much or more difcharge of the water, and confequently of declivity for the running off of the back-waters, may be loft, as has been gained by the feparation into different beds.

This difadvantage may be eafily remedied in thofe new channels and mouths of rivers which are dug by hands, and have fluices placed in them at the point of feparation from the original bed; for thefe fluices of communication need be opened only when there is a fuper-abundance of water in the river, fufficient to keep up the velocity in each of the channels; at other times they may be kept fhut, and the waters retained in their original bed.

52. It was for this purpofe, of preventing the damages proceeding from immoderate inundations, that the ancient Egyptians dug vaft lakes, and made fo many canals and fluices of communication between the Nile and thofe lakes, and from thence to the fea; that they might thereby be able to difcharge the waters into thofe refervoirs if they came down in too great abundance, or let them off again from thence upon the land, if the quantity of the natural inundation at any time was lefs than what was neceffary for the good of the country. By thefe means ancient Egypt was always mafter of its waters.

It is well known that it rains ſeldom in that country, and that the Nile by its regular inundations waters the land, by bringing down upon it the rains and melted ſnow from the high mountains of Abyſſinia. HERODOTUS[g] and DIODORUS SICULUS[h] have left us deſcriptions of the immenſe labours of the inhabitants to govern and multiply ſo beneficent a river, the particulars whereof are too well known to be repeated in this place. By theſe means Egypt became the granary of the world for above two thouſand years, and reimburſed, with immenſe advantage, the firſt expences.

RICCIOLI[i] aſſures us, that the ancient Perſians did the ſame thing with regard to the *Euphrates*, and for the ſame end. He adds, moreover, *Sic ubi Cyrus Gangem in Alveos* 460 *diſperſit, minora damna ex Gangis alluvionibus campi perpeſſi ſunt*; but I am totally at a loſs to find upon what authority he grounds this laſt aſſertion, for I never read that any CYRUS penetrated as far into India as the mouth of the *Ganges*, much leſs reigned ſo long over that country as to perform the vaſt work which RICCIOLI ſpeaks of.

(*g*) In lib. II.
(*h*) Biblioth. l. II. c. 1.
(*i*) Geogr. et Hydrogr. l. VI. cap. XXIX. p. 248, 249.

PLINY

PLINY [k] ſays, with regard to the different mouths of the Po, *Omnia ea flumina foſſaſque primi à Sagi fecere Thuſci, egeſto amnis impetu per tranſverſum in Atrianorum Paludes, quæ Septem Maria appellantur.* Theſe ſeven lakes diſcharged their waters into the ſea by ſeven mouths, which PLINY names in the ſame place. All this was apparently done that the river might do leſs damage to the adjacent countries by its frequent inundations. PLINY adds, *His ſe Padus miſcet, ac per hæc effunditur, pleriſque, ut in Ægypto Nilus, quod vocant Delta.*

To theſe examples, drawn from ancient hiſtory, might be added many modern ones, if the things in queſtion had need of further proofs. Thus, both nature and the experience of a long ſeries of ages teach us, that the ſeparation of a river into ſeveral beds, by new branches and mouths, is a means of diminiſhing inundations in the inner part of the country; but that this takes place only when there is a ſufficient abundance of water in the river to fill the new beds and channels ſo far as to prevent the velocity of the currents therein from being notably diminiſhed from what they were before the diviſion.

(*k*) Hiſt. Nat. l. III. cap. XVI.

SEC

SECTION II.

Laws of the meeting of oppoſite currents, with the application of them to ſluices.

53. When two equal currents of homogeneous fluids meet in oppoſite directions, there is firſt a ſwelling and riſing up of them at the point of rencounter; then follows a revulſion and counter current of each equally back again, ſo as to bring the whole to an equilibrium.

54. If the two oppoſite currents are unequal, either in force or in quantity, or in both, there will ſtill be a ſwelling and revulſion of each back again, but it will be diminiſhed in the greater current, and augmented in the leſſer, by the quantity by the which the one ſurpaſſes the other; and the point of rencounter of the two currents will have a flow and progreſſive motion in the direction of the ſtronger, the degree of velocity thereof being always in a direct ratio of the force and quantity of the one above the other.

55. If the fluids in oppoſite currents be not homogeneous, as is the caſe between ſea and river water, that which has the leaſt ſpecific gravity will ſwim upon the other, and continue to follow its firſt direction, until

ſuch

ſuch time as the heavier fluid ſhall have communicated its motion to all the parts of the lighter. But the lighter fluid will not loſe its former motion and direction at once, but *in a decreaſing ſeries, the law whereof will vary according to the greater or leſs difference of ſpecific gravity in the two fluids*, until the whole of the lighter has acquired the velocity and direction of the heavier which buoys it up.

The *time* and *ſpace* required for a greater current of ſalt water to communicate its motion and direction to an oppoſite one of freſh water will be but very little, ſince they differ in ſpecific gravity only $\frac{3}{73}$ parts that the ſalt is heavier than the freſh. It would require much greater between water and oil, and ſtill much more between quickſilver and oil, and ſo on. The elements for determining them in every caſe might be found by a proper number of experiments.

56. Let the two currents be equal or unequal in force and velocity *but nearly of the ſame ſpecific gravity*, if we ſhould ſuppoſe at the ſame time that their ſurfaces are not upon a level, but that the one is higher than the other (as is conſtantly the caſe in all ſluices that open to the ſea, except at the moment when the ſurface of the tide is upon a level with the ſurface of the water in the canal behind the ſluice); this circumſtance entirely changes both the caſe and the effects. It is certain, on

this

this ſuppoſition, that the overplus of velocity and elevation in the higher current, though it ſhould be the leſſer, will make the waters in the lower and greater current reflow upon themſelves until they come to a level and equilibrium with thoſe in the upper current; ſince theſe, by the laws of univerſal gravitation, cannot flow back from a lower to a higher level, but muſt deſcend according to the declivity of the ſurfaces. If the currents are of *very different ſpecific gravities*, they will come to an equilibrium according to the law laid down above (N° 55); but their greater or leſſer quantity and velocity will produce little or no effect in this caſe.

57. Now as the running of two currents in oppoſite directions, after their rencounter, and beyond the limits laid down above (N° 55.), is incompatible with, and contradictory to, the laws of nature, and conſequently impoſſible; we may draw this uſeful concluſion, which becomes important during inundations, and eſpecially during the annual overflowing of the low grounds in flat countries; to wit, that if the ſluices next the ſea againſt which the tide flows be ſhut only a quarter of an hour before the *flood* has riſen to the level of the water in the canal, not a drop of ſalt water can enter the ſaid canal, nor even into the ſluice itſelf; becauſe both the progreſſive motion of *the point of rencounter of*

the two currents, and the *over-swimming* of the fresh water upon the salt, will be always without the sluice and towards the sea, so long as the surface of the tide is below the level of the water in the canal. Many sluice masters, for want of knowing or considering this, are accustomed to shut their gates next the sea a little after *half flood*, under the pretence of preventing by this means the salt water from getting into the canal, and communicating thereby with the waters that overflow the low grounds in many places during winter, which would be of great detriment to the soil. Through this false persuasion, they lose no inconsiderable part of that time every day, which they might safely employ in drawing off the waters which overflow and incommode low and flat countries almost every winter and rainy season, as is the case in the Dutch and Austrian Netherlands.

SECTION IV.

Experiments to determine the different velocities, in different depths of water, of the same floating body moved uniformly by an equal force.

58. It is well known already, that for facilitating or retarding the motion of boats, &c. in canals, the different

depths of the water, above that fimply neceffary to keep them afloat, is a thing not at all indifferent. Dr. FRANKLIN has already treated this fubject, though perhaps not with fufficient accuracy, in a letter to Sir JOHN PRINGLE, written in the year 1769. He proves, however, that it is univerfally known among people accuftomed to work boats on canals, that there is a confiderable difference in the fwiftnefs of their motion according to the greater or lefs depth of the water therein; and that the water being low is of itfelf fufficient to retard the motion of a boat, without the keel thereof rubbing againft the bottom of the canal. The reafon he affigns for it is evident; for a boat cannot advance its own length in a canal without difplacing a quantity of water equal in mafs to the fpace which the boat occupies under the furface of the fluid. The water fo difplaced muft retrograde, and pafs under, and to the right and left, of the boat: fo that the lefs depth and breadth of water there is in the channel, the more in proportion it muft rife up and weigh againft the boat, and the more difficulty it muft find in paffing under and along fide of it, and neceffarily muft retard fo much the more the motion thereof. The refult of Dr. FRANKLIN's experiments on this fubject may be feen in the letter above mentioned.

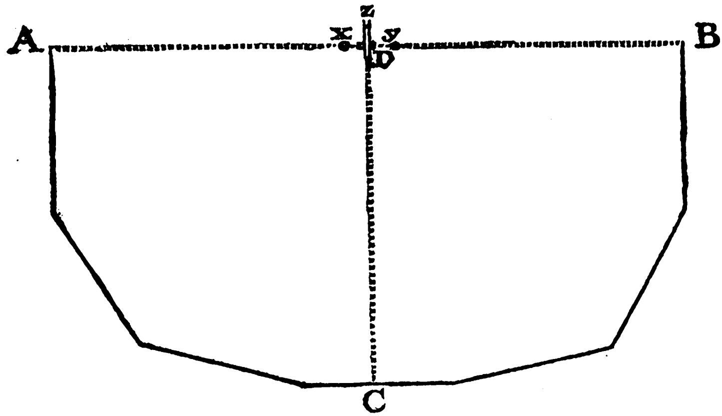
 59. Mr.

59. Mr. NEEDHAM, Director of the Imperial Academy of Sciences at Bruxelles, being of opinion that Dr. FRANKLIN's experiments were made upon too fmall a fcale to draw any very exact inferences from them, defired me, at the beginning of the year 1775, to make a new fet of experiments upon a much larger fcale and with all poffible exactnefs; I did accordingly, and fhall here give a fhort defcription of them.

I got made, by the fhip carpenters of Nieuport upon the Coaft of Flanders, an exact model of a bilander, anfwerable in all its proportions to thofe ufed in the Low Countries. Its length was thirty-nine Englifh inches, its breadth nine inches and a half; and its depth nine inches. Its form both within and without exactly reprefented that of a bilander. At each end of it was faftened perpendicularly a round and polifhed rod, ten inches and and a half in height above the fides of the boat.

I got made likewife a wooden canal, twenty feet in length, thirty-feven inches in breadth, and fixteen inches in depth; a fection of which is reprefented in the following figure:

This



This form is that of the excavation of the canals in the Low Countries, and approaches to that of the natural beds of rivers inafmuch as they are regular. Here AB = 37 Englifh inches, DC = 16 inches; and the length of the whole canal, as we faid before, was twenty feet. z reprefents the fection of a pulley fixed at one end of the canal, upon which paffed a fmall cord, one end of which was tied to the round rod at the fore part of the boat, and at the other end was a piece of lead which weighed eight ounces. This ferved for an equable force to give an uniform motion to the boat throughout all the experiments. *x* and *y* are fections of two other cords ftretched parallel to each other at about one inch and a half diftance, and reaching from one end of the canal to

2 the

the other. The two round rods fixed at the ends of the boat, moving within theſe parallel cords, ſerved to make the boat move in a right line in the middle of the canal, without running againſt either ſide, which it would have done without this precaution. The canal itſelf was upon an exact level, and one end of it, where the pulley was fixed, reſted upon the ſide of a well twenty-three feet deep, twenty of which were above the ſurface of the water; which gave ſufficient ſpace for the free and uniform deſcent of the lead-weight and cord running over the pulley, as they drew the boat from one end of the canal to the other.

Latts, exactly divided into inches, were nailed againſt each end of the canal within, to mark the different depths of the water in it according as it ſhould be augmented or diminiſhed. The outſides of the little boat, from its keel upwards, were likewiſe divided into inches. In the inſide of the boat was a quantity of ſand ſufficient to ſink it to ſix inches deep in the water. The common loaded bilanders in the Low Countries uſually draw ſix feet of water.

Thus the form of the wooden canal, together with its breadth and depth, and the form and dimenſions of the little boat, together with the depth of water it drew by means of its ballaſt of ſand, exactly correſponded with

thoſe

those in the real canals and bilanders in the Low Countries, an inch in the one anſwering to a foot in the other.

Cloſe to the canal, and out of the way of all wind, was ſuſpended a pendulum of fine waxed thread, to prevent the variations of the atmoſphere from altering its length, which from the point of ſuſpenſion to the center of gravity in the lead was $39\frac{1}{5}$ Engliſh inches, ſo that its iſochronic vibrations were exactly ſeconds of time.

60. It was neceſſary, in order to render the experiments exact, that they ſhould be made at a time when the air was perfectly calm; for the leaſt breath of wind, during the motion of the boat, cauſed great variations and irregularities in them, which it was abſolutely neceſſary to prevent, in order to be able to deduce any exact reſults from them. On the contrary, in a perfect calm, the times of the paſſage of the boat, from one end of the canal to the other, were exceedingly regular, as may be ſeen from the table of experiments which I give below.

By means of the pendulum I was able to meaſure the times of paſſage of the boat along the canal, in all the different depths of water, to a third or even to one quarter of a ſecond. The boat being held faſt againſt the back end of the canal by the hand of an aſſiſtant, and then let go, it was eaſy for me to perceive the preciſe

inſtant

inſtant of the beginning of its motion, to let go the pendulum at the ſame moment, and to count its vibrations till the inſtant that the boat ſtruck with an accelerated force againſt the fore end of the canal. As to the weight of eight ounces ſuſpended at the end of the cord, and which ſerved as *a moving force* to draw the boat along the canal, it was juſt as much as ſufficed to counterbalance the cord, and to put the boat in motion; leſs weight than that would do neither: therefore I was obliged to uſe ſo much, notwithſtanding the conſiderably accelerated motion it gave to the boat.

This is the whole mechaniſm of the inſtruments I uſed for the experiments in queſtion, and ſuch were the precautions I judged it neceſſary to uſe for making them with ſcrupulous exactneſs.

In the following table, which conſiſts of twelve columns, the firſt of them contains the different depths of water at which the experiments were made; the ten following ones contain ten different experiments made at each depth of water in the canal; and the twelfth or laſt column is the reduction of the ten others to a mean proportional or mean reſult of the whole, which is in *ſeconds of time*.

Table

Table of experiments made to aſcertain the times of paſſage of the boat along the canal, or its different degrees of velocity, in different depths of water.

Depths of Water.	1ſt Exp.	2d ditto	3d ditto	4th ditto	5th litto	6th ditto	7th ditto	8th ditto	9th ditto	10th ditto	Mean Reſults.
Inches 15	″ 14½	″ 14¾	″ 14⅗	″ 14½	″ 14½	″ 14⅗	″ 14⅗	″ 15	″ 14½	″ 14⅗	″ 14⅗
14	14¾	15	15	15½	15	15	14½	14½	15	15¼	15
13	15½	15½	15½	15¾	15½	15¾	15¾	15½	15¼	15¼	15½
12	16½	16	16	15½	16	15¾	16	16	16	16	16
11	16¾	17	17	16½	16¾	17	16¾	16½	16¾	16¾	16¾
10	17	18 wind	17	17½	17	17⅓	17½	17	17	17¾	17½
9	18½	18½	18	19	19	18	18	18¾	18	18½	18½
8	20	20	20½	20	20	20	20	19½	20	20	20
7	23½	23	22½	23	23	23	23½	23¼	23	23	23
6½	In the experiments made with this depth of water, the boat often touched the bottom.										30 by ſuppofition

It may be obſerved with regard to the laſt column of the above table, which contains the mean reſults or mean quantities of time which the boat takes to paſs from one end of the canal to the other in different depths of water, that it is given for the ſake of deſtroying thoſe little differences which are inevitable in practice; and it

 ſhews,

ſhews, as nearly as poſſible, what the true time of paſſage ought regularly to be when nothing happens to diſturb it.

It is alſo highly worthy of remark, that the *mean reſults* contained in this laſt column form a *ſeries of numbers regularly increaſing as the depths of water, wherein the reſpective experiments were made, regularly decreaſe*; ſo that *the different velocities of the floating body are in an inverſe ratio of the reſpective depths of the water in which it floats with an equal impulſive force, and that according to the law of the above ſeries.* This, perhaps, may furniſh elements to calculate, pretty near the truth, the different velocities of veſſels upon canals and rivers with different depths of water in all other caſes whatſoever. As to the concluſions to be drawn therefrom in practice, and in the common uſes of life, they are too obvious to need mentioning here.

SECTION V.

On the quantitity of declivity in rivers.

62. Abſtracting from all reſiſtance and friction, fluids, ſuch as water, deſcend upon planes let them be never ſo little inclined towards the center of the earth: and the

velocity of descent increases in a compound ratio of the increase of the mass of water, and of the greater declivity of the plane which serves for its bed (N° 13.).

63. Water, though unaffected by any compression or impulsion from above, cannot remain immoveable in any bed whatever except that which makes a curve perfectly concentrical with the *terrestrial curve*; but in this, being every where equally affected by the force of gravitation, it will remain without motion any way.

64. It follows from hence, that the springs and sources of all rivers must be at a greater distance from the center of the earth than one semi-diameter thereof, which is terminated at the surface of the sea; without which the waters could not run to the river's mouth.

65. Therefore, the absolute elevation of the surface of rivers is continually diminished as they recede from their springs, because of the necessary declivity of the beds of rivers towards the center of the earth; for without some degree of this declivity the waters could not run at all, as has been said above (N° 62. 63.).

66. The declivity of the beds of rivers cannot be a right line making a rectilinear angle with that horizontal plane which, being continued, would intersect their respective sources; but, if it is regular, it must be a curve which differs very little from that of the earth's surface,

and this, if the direction is in the parallels of latitude due Eaſt and Weſt, is *ſpherical*; but in all other directions it is a portion of an *oblate ellipſis*, on account of the earth's being a ſpheroid compreſſed by its axis. Now the horizontal plane which continued paſſes through the ſprings of rivers, is always a tangent to the curves of their beds at the point of inflexion, inſomuch as theſe are regular.

67. The quantity of abſolute declivity from the ſpring in any determinate part of a river, is that perpendicular line drawn from the point of greateſt current in that place till it meets *the curve concentrical to the earth's ſurface* which paſſes through the river's ſource. The declivity of the bed below the ſpring is had by taking the ſame perpendicular from the bottom of the bed; as that of the river's ſurface is by taking the perpendicular from thence.

68. If a plane be extended horizontally every way from the point of tangency to the earth's ſurface, or from the point where it is perpendicular to any radius of the earth, water will run from every other part of the plane towards that ſaid point which is nearer to the center of the earth than any other point in the whole plane.

69. The depreſſion of *the curve of a river's bed*, below *the concentrical-terreſtrial-curve which interſects its ſource*,

source, being only 250 fathom perpendicular in a course of 500 leagues, it will be sufficient to give a notable current in a regular bed throughout all that extent of river, as appears from what we have said above (N° 38.). But the depression of *the curve of the river's bed*, below *the horizontal plane which is a tangent to its source*, in this same extent of course, is not less than ninety leagues perpendicular, being always *the secant of the arc of the river's extent minus a radius of the earth in that point.*

70. It follows evidently from the above princples (N° 62—69), that the declivity and velocity of a river are less in proportion as the bed approaches nearer to being concentrical with the curve of the earth's surface.

71. I shall now apply the principles laid down to determine, as near as possible, the real quantity of declivity in different rivers, making use of what is already known from experiments and actual mensuration to determine the same in all others by the comparison of the different degrees of velocity in their respective currents.

72. It is the general opinion of most of those who have examined this subject[k], that rivers and canals which have less than one foot of declivity in 10,000 feet of course, will have very little current, unless it be by means of the great abundance of their upper waters

(*k*) Vide DESCHALES, de Font. et Fluv.

which give motion to thofe before them by their weight and impulfion. Without this the refiftance proceeding from the bottom and fides of the bed, and from other accidental obftacles (N° 21.) would equal, if not furpafs the ordinary caufes of acceleration (N° 20) fo as to diminifh continually the motion of the waters, and at laft render them almoft ftagnant (N° 22.). But nature has prepared remedies againft this, as we have feen above (N° 28. 29. 48.). What RICCIOLI (l) fays of the *Po*, in that part of its courfe next its mouth, is perfectly conformable to this theory: " Sic Padus, qui a Pago *Oftellata* " vocato, ufque ad *Adriaticum*, intervallo milliarium cir- " citer 70, non habet libramentum majus 13 aut 14 " pedum, ita ut fingulis milliaribus ne 3 quidem unciæ " declivitatis obveniant; unde *Padufæ*, potius inftar ftag- " nantibus aquis, incertiffimus effet ad defluxum curfus: " impetu tamen impreffo à 30 et amplius fluminibus " aut torrentibus fe in illum exonerantibus, etiamque a " nativæ pondere aquæ ex fuperioribus et altioribus " prope Alpes alveis decurrentis, velocitatem maximam " acquirit."

73 From many obfervations and trials which I made for this purpofe in the years 1773 and 1774 upon the river Iprelee in Flanders, which comes down from the

(l) Geogr. et Hydrogr. l. VI. c. XIX. p. 215. edit. Bonon. 1661.

city

city of Ipres and falls into the ſea at Nieuport, having *a very moderate current* when the ſluices upon it are open, I found its *mean declivity* to be nearly three fathoms four feet and eight inches in 20,000 fathoms of extent of its courſe, or very nearly one foot in a meaſured Engliſh mile. I ſay its *mean declivity*, becauſe from what has been ſaid above (N° 13. 27. 28. 29.) it is plain, that a greater or leſs quantity than ordinary of water in it will add to, or take from, ſomething thereof; but the declivity in each part of its bed is nearly uniform.

As the ſources of this river, and thoſe of the Iſere which joins it at Fort Knock, four leagues from Nieuport, are in the higher grounds of Flanders towards Houthem, Mount Kemele, Swaertſberg, Catſberg, and the other hills as far as Mount Caſſel; and as the reſt of their courſe is in a flat country with a very ſmall deſcent towards the ſea, the declivity thereof may be taken as *a mean* between that of the other rivers and canals of Flanders: the *artificial canals* will have leſs, not above a ſix or ſeven thouſandth part of their extent, or one twelfth of an inch in each eight fathoms: the rivers Lys and Eſcaut, before they fall into the flat country, ſomething more, after which they may have about the ſame, or even ſomething leſs between Ghent and Antwerp.

This

This quantity of three fathoms four feet and eight inches of declivity in 20,000 fathoms of extent, gives the proportion of the declivity to the extent as 1 to 5292, which is one line or twelfth part of an inch in $6\frac{1}{8}$ fathoms, and two feet ſeven inches in one French league of 2283 fathoms. Now the meaſured Engliſh mile containing 5280 feet, this proportion of $\frac{1}{5292}$ approaches ſo very near to one foot of declivity in every meaſured mile of extent, that I ſhall every where reduce what I call the *mean declivity* to that quantity, as a ſtandard wherewith to compare the reſt.

74. In canals, all whoſe ſluices and vents have been kept ſhut a ſufficient time to render the water ſtagnant throughout their whole length, there cannot be allowed above an inch or two of declivity for each mile in length, on account of the water that unavoidably runs off through the chinks of the doors of ſluices, drains, &c.

75. According to the obſervations of the Abbe CHAPPE D'AUTEROCHE [(m)], the floor of the Hall of the Royal Obſervatory at Paris is forty-five fathoms three feet and five inches French above the level of the ſea at the mouth of the Seine. According to the Abbé NOLLET, this ſame floor is forty-ſix fathom above the level of the Ocean, and only forty-five fathoms above the level of the Mediter-

(*m*) See Relation de ſon Voyage en Siberie, tom. II. p. 406, 407. 444.

ranean

ranean ſea. Again, according to the above Abbé CHAPPE, the ſaid floor of the Obſervatory is elevated twenty-four fathoms one foot and ten inches above the level of the river Seine at Paris; therefore the level of the Seine under the Pont Royal at Paris is twenty-one fathoms one foot and ſeven inches above the level of the Ocean; and ſuch alſo is the quantity which Meſſ. CASSINI have given, from their own obſervations and experiments, for the mean height of the Seine at Paris above the level of the ſea.

Now the courſe of the Seine from Paris to its mouth at Havre de Grace, by following all its turns and windings, is about 90,000 fathoms in length; therefore $\frac{90,000}{21.1.7} = 4232\frac{1}{2}$ fathoms of extent for one fathom of declivity in the river Seine, or one line in $4\frac{2}{3}$ fathoms, and conſequently the proportion of its declivity to its length is as one to $4232\frac{1}{2}$. It is to be obſerved, that the bed of the Seine is deep, and its current conſiderably ſtrong.

76. By ſimilar obſervations and actual levellings made upon the river Loire by M. M. PICARD and PITOT [n], the declivity thereof in proportion to its length is found to be as one to 3174, which is one line in $3\frac{2}{3}$ fathoms. Notwithſtanding this great declivity of the bed of the Loire it is obſerved, that the velocity of the water therein,

(n) See Memoires de l'Acad. Royale des Sciences de Paris, pour 1730.

 compared

compared to that in the Seine, is leſs than it ſhould be in proportion to their reſpective declivities, which is very juſtly attributed to the much greater depth in proportion to the breadth of the Seine, above what is found in the Loire. This laſt river is remarkably broad, and ſo ſhallow that in many places it is hardly navigable for boats. Now this, according to the principles laid down above (N° 27, &c.) muſt very much diminiſh the ſwiftneſs of the current, which it ſhould otherwiſe have from the great declivity of its bed. In confirmation of this it is moreover obſerved, that in great falls of rain, which equally increaſe the volume of water in both theſe rivers, the velocity in the Loire augments in a much greater proportion than it does in the Seine; and this obſervation is likewiſe conformable to the principles above laid down (N° 12. 28. &c.).

77. The river Doux, after paſſing by Beſançon, falls into the Saone above Chalon; the Saone joins the Rhône at Lyons. This river, from Beſançon to its mouth in the Mediterranean ſea, is one of the moſt rapid in the known world: the velocity of its current is at leaſt double to that of the Seine or Loire, and its courſe is almoſt in a ſtraight line. The difference of elevation of this river at Beſançon, above that of its mouth in the Mediterranean ſea, after a courſe of about eighty-ſix French leagues, has been

been found, by a long ſeries of barometrical obſervations, to be about ſeventy-five fathom [a], which gives the proportion of the declivity to the extent as one to 2620, or about one third of a line to each fathom. This is double the *mean declivity* of the rivers in Flanders; but the velocity of the current in the Rhone is at leaſt triple that in the others (N° 29.).

78. From the above *data*, got from obſervations and actual menſuration, and from many others of the ſame nature too long to mention here, we may deduce the following table of comparative proportions between the declivities and velocities in different kinds of rivers.

(a) See Cours de Phyſique de Para, tom. II. N° 740.

Rates or classes of rivers and flowing waters.	Comparative degrees of the mean velocities of currents	Seconds of time wherein currents run 20 fathom.	Fathom run by the current in 1 minute of time.	Ratios of declivity compared with horizontal length.	Fathoms of length for each $\frac{1}{12}$ inch of declivity.	Distinctive attributes of the various kinds of rivers and flowing waters.
1	0	0	0	$\frac{1}{12000}$	14	Channels wherein the resistance from the bed, and other obstacles, equal the quantity of current acquired from the declivity; so that the waters would stagnate therein, were it not for the compression and impulsion of the upper and back-waters.
2	$\frac{2}{3}$	180	$6\frac{2}{3}$	$\frac{1}{7000}$	8	Artificial canals in the Dutch and Austrian Netherlands.
3	1	120	10	$\frac{1}{5200}$	6	Rivers in low and flat countries, full of turns and windings, and of a very slow current, subject to frequent and lasting inundations.
4	$1\frac{1}{2}$	80	15	$\frac{1}{4000}$	$4\frac{2}{3}$	Rivers in most countries that are a mean between flat and hilly, which have a good current, but are subject to overflow: Also, the upper parts of rivers in flat countries.
5	$2\frac{1}{6}$	55	$21\frac{2}{3}$	$\frac{1}{3200}$	$3\frac{2}{3}$	Rivers in hilly countries, with a strong current, and seldom subject to inundations: Also, all rivers near their sources have this declivity and velocity, and often much more.
6	3	40	30	$\frac{1}{2600}$	3	Rivers in mountainous countries, having a rapid current and straight course, and very rarely overflowing.
7	5	24	50	$\frac{1}{2000}$	$2\frac{1}{3}$	Rivers in their descent from among mountains down into the plains below, in which places they run torrent-wise.
8	8	15	80	$\frac{1}{1700}$	2	Absolute torrents among mountains.

I should

I ſhould think it quite ſuperfluous to give any explanation of a table ſo clear and intelligible as the above; and ſhall only remark upon it that *the comparative degrees of the mean velocities of the reſpective currents* in the ſecond column are the reſult of obſervations and experiments, the method of making which has been given above (N° 26.): but as the velocity of rivers is very different in different ſeaſons of the year, which augment or diminiſh greatly the maſs of waters in their beds, *a mean* has been kept to, as much as poſſible, in the above table.

By taking the degree of velocity of the current in any river, a thing ſo eaſy to be done; and obſerving its other characteriſtics as laid down above under the title of *diſtinctive attributes*, it will be eaſy to judge very nearly of *the quantity of declivity in that part of the river*.

79. After carefully comparing what has been ſaid in the relations of travellers, and in the beſt treatiſes of geography, upon the principal rivers in the known world, I ſhould be inclined to claſs them in the following manner, particularly entreating at the ſame time that my opinion about it may be regarded as ſimple conjecture, which I leave to be rectified by thoſe better acquainted with the matter than it is poſſible for me to be.

Under the firſt rate or claſs in the above table I ſhould put that part of the channel of moſt great rivers which is

is in extensive plains next the sea; with regard to *the declivity thereof alone*, but not at all with regard to the velocity of the current there, which is often very great from the compression and impulsion of the upper waters, as has been repeatedly shewn above must be the case (N° 29. 38. 43. 72.).

Second rate or class. Most artificial canals in flat countries, made for the use of navigation; especially those in the Dutch and Austrian Netherlands.

Third rate or class. The river Trent; the Scheld and the Lys below Ghent; the Isere and the Iprelee below Fort Knock in Flanders; many rivers in the territories of Bologna and Ferrara in Italy; the river Meander in Natolia; and innumerable others in flat countries.

Fourth class. The Thames; the Lys and the Scheld above Ghent in Flanders; the Senne, the Dyle, and the Demmer, in Brabant; the Seine and the Somme in France; the Nile and the Niger in Africa; the rivers of St. Lawrence below Lake Ontario, the Orounoko, the river of Amazons, and the rivers of Paraguay, in America.

Fifth class. The Severn and Ouse in England; the Loire and Garonne in France; the Tagus, the Guadiana, and the Guadalquivir, in Spain; the Po and the Tiber in Italy; the Meuse, the Rhine, and the Elbe, in Germany; the Weissel, the Neister, the Bog, and the Nieper,

in

in Poland; the Don and the Dwina in Ruſſia; the Amur or Saghalien in Tartary; the Yellow and Blue Rivers in China; the rivers of Cambodia, Ava, and Ganges, in India; the Euphrates; the river Zaire in Congo; the Miſſiſippi.

Sixth claſs. The Rhone in France; the Ebro and Douro in Spain; the Danube; the Wolga; the Irtiſch and Oby, the Jeneſea and Lena, in Siberia; the river Indus; the Tigris; the Malmiſtra in Cilicia.

Seventh claſs. In this claſs can only be enumerated thoſe parts of rivers where they deſcend from among mountains into the plain country below; as alſo ſome rivers paſſing through the midſt of mountains.

Eighth claſs. To this claſs belong all torrents among mountains; ſuch, for example, as the Bourns in the Highlands of Scotland are deſcribed to be.

SECTION VI.

A general and eaſy method of taking levels through large extents of country where rivers paſs; and alſo of computing the heights of interior parts of continents above the ſurface of the ſea.

80. After all I have ſaid hitherto in this eſſay, and particularly in the foregoing ſection, what I am about to

lay

lay down under this last head of it must appear very plain and easy. I am very far, however, from giving the methods I am going to propose *for taking the levels through whole countries and continents as far as rivers extend*, as strictly exact; I know very well that it is next to impossible they should be so, considering the continual variations in the declivities of rivers, and in the velocities of their currents in different parts, as also the impossibility of knowing the exact length of their course through all their turns and windings I only give them therefore as a general and easy method of computing the relative heights of countries without deviating much from the truth, which, perhaps, is all that may be necessary for the consideration of the natural philosopher. At all events, they may be of some use, for this end, in so many parts of the earth through which rivers pass, and where no barometrical observations, or any others whatever, for taking heights above the sea, have been, or perhaps ever will be made. They may also be found useful in taking the levels through a large extent of flat countries where regular canals and rivers pass, and where the difference of elevation is too small to be observed by the barometer, and where also the taking them through so great an extent by the common methods of levelling would be much too expensive for the purposes required. Now in

this laſt caſe I have found, by experience, that by the method I here propoſe the difference of heights may very eaſily be found, and that very near to the truth.

For this end it may be proper to premiſe a few neceſſary conſiderations and precautions to be obſerved in making uſe of the method I here propoſe. They would eaſily occur to any one who conſiders the principles whereon it is grounded; but to ſave trouble I ſhall put them down in a few words.

81. The firſt is, that a particular attention muſt be had to the quantity of water actually in the river at the time of the operation, ſo that according as the greater or leſs quantity thereof may augment or diminiſh the velocity of the current, allowance may be made conformable thereto in determining the quantity of declivity from the degree of velocity.

2dly, Obſerving this precaution throughout the whole river, or all that part of it wherein we want to find the difference of elevations, we muſt next endeavour to determine, as near as poſſible, by the principles laid down in the laſt ſection, *all the variations of declivity from the variations of velocity within thoſe limits, and alſo the exact length and quantity of each.*

3dly, The ſame attention muſt be had in taking the difference of heights by canals, while their ſluices and,

 com-

communications are kept conſtantly open, ſo as to effectuate a compleat natural current throughout the whole extent thereof; for in this caſe they are no other than rivers, and their waters follow the ſame laws of motion.

4thly, But in canals which are ſhut, and their waters kept up by ſluices ſo as to render them nearly ſtagnant, the practice of this method will be different from what it is in rivers and open canals: for in this caſe there cannot be allowed for the declivity of the ſurface of the water from ſluice to ſluice above one inch, or two at moſt, in each mile of length, according as there may be fewer or more accidental drainings of the water in it (N° 74.).

Again, as it may happen, in taking the levels of countries by the means of artificial canals, that the water in different parts may have different directions, attention muſt be had to *add* or *ſubtract* reſpectively the total declivity of each.

Moreover, it almoſt always happens, in canals where the ſluices are ſhut, that the water on the two ſides of each ſluice is of a very different height, the back waters being kept up, while the lower are run off to a certain point; but in ſluices next the ſea, the tide againſt the outer gates is ſometimes lower and ſometimes higher than the water in the canal above. In all theſe caſes, the difference of height muſt be exactly meaſured, and the

quantity

quantity respectively added or subtracted in the account of the levelling.

5thly, After this it is necessary to determine, as nearly as possible, the length of the canals and rivers through all their turns and windings, and throughout the whole extent of country in which we want the difference of elevations. This may be done by an actual mensuration, or by the general opinion of the inhabitants of each part of the country, which, being founded upon the long and continually repeated experience of an infinity of people, will be found to differ very little from the truth, *attention being had to the quantity of their nominal measures*; even the errors in *more* or *less* will nearly compensate each other; or, finally, in great extents it may suffice to compute them from good geographical maps.

6thly, This being done by one or the other of these methods, it will be easy, from the quantity of declivity before determined for each part in particular, to find the whole quantity of declivity throughout the whole extent of country measured, or from any one part thereof to any other along the rivers or canals in question, which are supposed to be continued without interruption from one place to the other. If to this be added the relative height of the country in each place compared with the level of the water in the part of the river or canal next to each,

we fhall have very nearly the difference of elevation of thofe two parts of the country. And thus the levels may be taken from the fea through any extent of country, nay even through whole continents, as far as rivers or canals extend without interruption. Cataracts themfelves, fuch as thofe in the Nile and in the river of St. Lawrence, need not hinder the operation, fince we have only to take the refpective heights from which they fall into the account as we do in common fluices, and allow for the increafe of velocity produced by them in the current of the river above and below the places where they exift.

82. Although I do not pretend to equal this method (of finding the difference of heights in countries) *for exactnefs* to the levels taken by actual menfuration, or to thofe found by a long feries of nice barometrical obfervations; yet it muft be allowed, that it is free from many inconveniencies, and accompanied with many conveniences, which the others are not. It may be eafily carried through great extents of country, where the other methods cannot be put in practice, on account of the expence or time required; and this may be done with very little trouble, and perhaps with fufficient exactnefs to anfwer all the purpofes of the natural philofopher in his confiderations on the globe we inhabit. Although the method of taking heights by barometrical obfervations

obſervations is highly uſeful, and among mountains (where mine can be of little or no ſervice) far preferable to every other hitherto diſcovered; yet it will eaſily be acknowledged by every one who is acquainted with what M. DE LUC (m), Sir GEORGE SHUCKBURGH, and Colonel ROY (n), have done upon this ſubject, that the greateſt attention to an infinity of varying circumſtances, as well as the greateſt nicety and exactneſs both in the inſtruments and in repeated obſervations, are neceſſary if we would come at the truth thereby.

Again, the method of taking the difference of heights by the quantity of declivity in rivers requires no attention to the curvature of the globe, an object (as every one knows) infinitely too conſiderable to be neglected in the common method of levelling; as are alſo the great and varying refractions of the viſual rays ſo near to the ſurface of the earth as they muſt be taken in the practice of that method. The quantity of effects and of errors in the viſuals proceeding from this laſt cauſe muſt be very different at different times, as it depends wholly on the greater or leſs denſity, on the greater or leſs quantity of vapours ſuſpended in the loweſt part of the atmo-

(m) See his work on Barometers and Thermometers, in two vol. quarto.

(n) See the learned and curious treatiſes of theſe two gentlemen in the Philoſophical Tranſactions for 1777.

ſphere,

ſphere, the ſtate of which ſeldom remains long the ſame. Now it is no eaſy matter either to determine the quantity of theſe exactly, or to calculate the effects and errors in the viſual rays proceeding therefrom, which yet muſt be done to come at the truth by the common method of levelling; whereas, in the method I propoſe, no ſuch conſiderations are neceſſary, as is evident from the nature of it.

But this is more than enough on a method ſo obvious and eaſy; I ſhall now give a few examples of it, and thereby conclude this eſſay, already perhaps much too long.

83. Suppoſing the length of the Scheld, between Antwerp and Ghent, following all its meanders, to be forty meaſured Engliſh miles, as it is reckoned nearly to be; and ſuppoſing the length of the ſaid Scheld between Ghent and Tournay to be fifty of the ſame miles; and that of the Lys from Ghent to Commines, where it approaches neareſt to the city of Ipres, forty-ſix miles; it is required to know the reſpective differences of elevation between all theſe places.

It may be found above (N° 78. 79.) that the river Scheld, between Ghent and Antwerp, has not above one foot declivity in each mile of its courſe; and that the

Scheld and the Lys, above Ghent, have about one foot declivity in each four thoufand feet of length.

According to this, the furface of the Scheld in Ghent is about forty feet higher that it is at Antwerp; and at Tournay it is fixty-fix feet higher than at Ghent, and one hundred and fix feet higher than at Antwerp. So alfo the furface of the Lys at Commines is fixty-one feet higher than at its junction with the Scheld in Ghent, and one hundred and one feet higher than the fame at Antwerp. From hence it may be deduced, that the Scheld at Tournay is about five feet higher than the Lys at Commines, through twenty-five miles of interjacent country.

84. Suppofe it be required to find the difference of height between the furface of the Lys at Commines and the furface of the canal at Ipres which falls into the fea at Nieuport on the Coaft of Flanders. The diftance between Ipres and Commines is nearly feven meafured miles, through which there is no communication by water; but there is one a great way round, which therefore, for the purpofe required, muft be followed through all the differences of elevation comprized therein, *viz.*

Defcent

Descent towards the sea at Nieuport *on the Coast of* Flanders.

	Feet.
Total declivity of the Lys from Commines to Ghent, 46 miles, - - - - -	61
Declivity of the canal from Ghent to Bruges, when the sluices are shut, - - -	1
Difference of height of the water in the aforesaid canal above that in the canal from Bruges to Ostend, which two communicate together by sluices, - - - -	8
Declivity from Bruges to Plaschendahl where the canal of Nieuport joins that of Ostend, -	6
Total declivity from Plaschendahl to Nieuport, including the difference of surfaces in two intermediate sluices, - - - - -	8
Total declivity, all one way, from Commines to Nieuport, about 95 miles, - -	84

Ascent

too far into the regions of conjecture; but as ſuch miſtakes as theſe are no ways prejudicial to my fellow creatures, to whom I wiſh to be uſeful, and as they may give occaſion for others to rectify them, and ſo lead them to a ſubject which otherwiſe, perhaps, they might never have attended to, I ſhall hope for indulgence from all thoſe who wiſh well to humanity and to uſeful knowledge.

from what was found by actual levels made from the Lys to Ipres by the French engineers during the time that LEWIS the XIVth was maſter of the country, when there were propoſals for opening a canal from the one to the other.

85. I ſhall venture to carry my conjectures ſtill farther, and grounding them upon the principles laid down above (N° 78. 79.) I ſhall take a general view of the elevations of continents along the courſe of the principal rivers in the known world. I cannot, however, repeat too often, that I give this as a matter of mere conjecture and curioſity. It has not been, nor ever will be, in my power, or in that of any other particular perſon whatſoever, to follow the courſes of all the rivers mentioned in the enſuing table from their mouths to their ſources. All that can poſſibly be done on this head, is to examine the relations of voyagers and geographers concerning each river as far as it is known, and to reduce it by that means within the compaſs of the hydrometrical principles laid down in this eſſay. This is what I have done as far as I could; and therefore, allowing that I make great miſtakes therein, yet I do not think that I merit much blame on that account, as I have done what I was able to do. If I am blame-worthy, it is for having launched out

Ascent from the sea at Nieuport *to the city of* Ipres.

	Feet.
Difference of height which the river from Nieuport to Ipres has above the canal from Nieuport to Plaſchendahl, taken croſs the harbour of Nieuport, by means of the tides which come up againſt the outer gates of the ſluices next the ſea on each,	3
Declivity of the river Iprelee from Nieuport to Boeſinghe, - - - -	11
Difference of level of the water above and below the ſluice of Boeſinghe, - -	$22\frac{2}{3}$
Declivity in the canal from Boeſinghe to Ipres, -	$\frac{1}{3}$
Total aſcent, all one way, from Nieuport to Ipres,	37

Now 84−37=47 feet for the difference of height which the ſurface of the Lys at Commines has above the ſurface of the canal at Ipres. I have made uſe of the above example preferably to any others, as it is very complicated, and becauſe the quantities of declivity which I have put down are not arbitrary; and, moreover, becauſe I had the good fortune to find, ſome years after I had taken thoſe meaſures, that I only differed two feet

 from

86. A table of the elevation of countries above the ſurface of the ſea, at each 100 miles of length up the courſe of the principal rivers in the world, as far as they extend; by computation from the principles laid down in this treatiſe.

Feet of elevation in 100 miles.	Feet of elevation in 200 miles.	Feet of elevation in 300 miles.	Feet of elevation in 400 miles.	Feet of elevation in 500 miles.	Feet of elevation in 600 miles.	Feet of elevation in 700 miles.	Feet of elevation in 800 miles.	Feet of elevation in 900 miles.	Feet of elevation in 1000 miles.	Names of Rivers, And quantities whereby the length of their courſe is to be diminiſhed, to have the diſtance from their mouths in a direct line.
100	210	330	470	630	820	1040	1300	1600	1950	The river Trent, the Meander, and many others of the ſame kind, which are ſeldom of great extent: in theſe one-third of the courſe may be allowed for its deviations from a right line.
150	310	480	670	880	1120	1400	1770	2160	2620	The river Thames; the Seine and the Somme in France; the Nile and the Niger in Africa; the river St. Laurence, the Oroonoko, the river of Amazons, the rivers of Paraguay: in theſe about one-fourth of the length of courſe may be allowed for turns and windings in it.
220	450	700	980	1290	1640	2040	2500	3030	3630	The Severn; the Loire and Garonne in France; the Tagus, Guadiana and Guadalquivir in Spain; the Po and the Tyber in Italy; the Meuſe, Rhine and Elbe in Germany; the Weixel, Neiſter, Bog and Nieper in Poland; the Don and Dwina in Ruſſia; the Amur in Tartary; the Hoang-ho-keou and Yang-tſe Kiang-keou in China; the rivers of Cambodia, Ava and Ganges in India; the Euphrates; the Zaire in Congo; the Miſſiſippi: in theſe may be allowed about one-fifth of the length of the courſe for turns and windings in it.
300	650	1050	1520	2070	2720	3500	4440	5570	6920	The Rhône in France; the Ebro and Douro in Spain; the Danube; the Wolga; the Irtiſch, Oby, Jeneſca and Lena in Siberia; the Malmiſtra in Cilicia; the Tigris; the Indus. The courſe of theſe rapid rivers is uſually very ſtraight, and there cannot be above one-ſixth of the length thereof allowed for deviations from a right line.